

Joram Lindenstrauss Lior Tzafriri

Classical Banach Spaces I

Sequence Spaces

Reprint of the 1977 Edition

Classical Banach Spaces II

Function Spaces

Reprint of the 1979 Edition



Springer

Joram Lindenstrauss
Lior Tzafriri
Department of Mathematics
The Hebrew University of Jerusalem
Jerusalem 91904
Israel

Originally published as Vol. 92 and Vol. 97 of the
Ergebnisse der Mathematik und ihre Grenzgebiete

Cataloging-in-Publication Data applied for

Die Deutsche Bibliothek - CIP-Einheitsaufnahme

Lindenstrauss, Joram:

Classical Banach spaces / Joram Lindenstrauss ; Lior Tzafriri. -
Reprint of the 1977, 1979 ed. - Berlin ; Heidelberg ; New York ;
Barcelona ; Budapest ; Hong Kong ; London ; Milan ; Paris ;
Santa Clara ; Singapore ; Tokyo : Springer, 1996
(Ergebnisse der Mathematik und ihrer Grenzgebiete ; Vol. 92 und 97)
(Classics in mathematics)
Enth.: 1. Sequence spaces. 2. Function spaces

NE: Tzafriri, Lior:; 1. GT

Mathematics Subject Classification (1991):

46-02, 46A40, 46A45, 46BXX, 46JXX

ISBN 978-3-540-60628-4 ISBN 978-3-540-37732-0 (eBook)
DOI 10.1007/978-3-540-37732-0

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustration, recitation, broadcasting, reproduction on microfilm or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provision of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer-Verlag. Violations are liable for prosecution under the German Copyright Law.

© Springer-Verlag Berlin Heidelberg 1996

Originally published by Springer-Verlag Berlin Heidelberg New York in 1996.

The use of general descriptive names, registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

SPIN 10485236 41/3144- 5 4 3 2 1 0 – Printed on acid-free paper

Joram Lindenstrauss Lior Tzafriri

Classical Banach Spaces I

Sequence Spaces



Springer-Verlag Berlin Heidelberg GmbH

**Joram Lindenstrauss
Lior Tzafriri**

**Department of Mathematics, The Hebrew University of Jerusalem
Jerusalem, Israel**

AMS Subject Classification (1970): 46-02, 46 A45, 46Bxx, 46Jxx

ISBN 978-3-540-60628-4 ISBN 978-3-540-37732-0 (eBook)
DOI 10.1007/978-3-540-37732-0

Library of Congress Cataloging in Publication Data. Lindenstrauss, Joram, 1936-.
Classical Banach spaces. (Ergebnisse der Mathematik und ihrer Grenzgebiete; 92).
Bibliography: v. H1, p. Includes index. **CONTENTS:** 1. Sequence spaces. 1. Banach spaces.
2. Sequence spaces. 3. Function spaces. I. Tzafriri, Lior, 1936- joint author. II. Title. III.
Series QA322.2.L56, 1977- 515'.73. 77-23131.

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically those of translation, reprinting, re-use of illustrations, broadcasting, reproduction by photocopying machine or similar means, and storage in data banks. Under § 54 of the German Copyright Law where copies are made for other than private use, a fee is payable to the publisher, the amount of the fee to be determined by agreement with the publisher.

© by Springer-Verlag Berlin Heidelberg 1977.

Originally published by Springer-Verlag Berlin Heidelberg New York in 1977.

Typesetting: William Clowes & Sons Ltd., London, Beccles and Colchester.

2141/3140-543210

Table of Contents

1. Schauder Bases	1
a. Existence of Bases and Examples	1
b. Schauder Bases and Duality	7
c. Unconditional Bases	15
d. Examples of Spaces Without an Unconditional Basis	24
e. The Approximation Property	29
f. Biorthogonal Systems	42
g. Schauder Decompositions	47
2. The Spaces c_0 and l_p	53
a. Projections in c_0 and l_p and Characterizations of these Spaces	53
b. Absolutely Summing Operators and Uniqueness of Unconditional Bases	63
c. Fredholm Operators, Strictly Singular Operators and Complemented Subspaces of $l_p \oplus l_r$	75
d. Subspaces of c_0 and l_p and the Approximation Property, Complementably Universal Spaces	84
e. Banach Spaces Containing l_p or c_0	95
f. Extension and Lifting Properties, Automorphisms of l_∞ , c_0 and l_1	104
3. Symmetric Bases	113
a. Properties of Symmetric Bases, Examples and Special Block Bases	113
b. Subspaces of Spaces with a Symmetric Basis	123
4. Orlicz Sequence Spaces	137
a. Subspaces of Orlicz Sequence Spaces which have a Symmetric Basis	137
b. Duality and Complemented Subspaces	147
c. Examples of Orlicz Sequence Spaces	156
d. Modular Sequence Spaces and Subspaces of $l_p \oplus l_r$	166
e. Lorentz Sequence Spaces	175
References	180
Subject Index	185

Joram Lindenstrauss Lior Tzafriri

Classical Banach Spaces II

Function Spaces



Springer-Verlag Berlin Heidelberg GmbH 1979

Joram Lindenstrauss
Lior Tzafriri

Department of Mathematics, The Hebrew University of Jerusalem
Jerusalem, Israel

AMS Subject Classification (1970): 46-02, 46 A40, 46Bxx, 46Jxx

ISBN 978-3-662-35349-3 ISBN 978-3-662-35347-9 (eBook)
DOI 10.1007/978-3-662-35347-9

Library of Congress Cataloging in Publication Data. Lindenstrauss, Joram, 1936-. Classical Banach spaces. (Ergebnisse der Mathematik und ihrer Grenzgebiete; 92, 97). Bibliography: v. 1, p. ; v. 2, p. Includes index.
CONTENTS: I. Sequence spaces. 2. Function spaces. 1. Banach spaces. 2. Sequence spaces. 3. Function spaces. I. Tzafriri, Lior, 1936- joint author. II. Title. III. Series QA322.2.L56. 1977-515'.73. 77-23131.

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically those of translation, reprinting, re-use of illustrations, broadcasting, reproduction by photocopying machine or similar means, and storage in data banks. Under § 54 of the German Copyright Law where copies are made for other than private use, a fee is payable to the publisher, the amount of the fee to be determined by agreement with the publisher.

© by Springer-Verlag Berlin Heidelberg 1979.

Originally published by Springer-Verlag Berlin Heidelberg New York in 1979.
Softcover reprint of the hardcover 1st edition 1979

2141/3140-543210

Table of Contents

1. Banach Lattices	1
a. Basic Definitions and Results	1
Characterizations of σ -completeness and σ -order continuity. Ideals, bands and band projections. Boolean algebras of projections and cyclic spaces.	
b. Concrete Representation of Banach Lattices	14
Abstract L_p and M spaces. Joint characterizations of $L_p(\mu)$ and $c_0(\Gamma)$ spaces. The functional representation theorem for order continuous Banach lattices. Köthe function spaces. The Fatou property.	
c. The Structure of Banach Lattices and their Subspaces	31
Property (u). Weak completeness and reflexivity. Existence of unconditional basic sequences.	
d. p -Convexity in Banach Lattices	40
Functional calculus in general Banach lattices. The definition and basic properties of p -convexity and p -concavity. Duality. The p -convexification and p -concavification procedures. Some connections with p -absolutely summing operators. Factorization through L_p spaces.	
e. Uniform Convexity in General Banach Spaces and Related Notions	59
The definition of the moduli of convexity and smoothness. Duality. Asymptotic behavior of the moduli. The moduli of $L_2(X)$. Convergence of series in uniformly convex and uniformly smooth spaces. The notions of type and cotype. Kahane's theorem. Connections between the moduli and type and cotype.	
f. Uniform Convexity in Banach Lattices and Related Notions	79
Uniformly convex and smooth renormings of a ($p > 1$)-convex and ($q < \infty$)-concave Banach lattice. The concepts of upper and lower estimates for disjoint elements. The relations between these notions and those of type, cotype, p -convexity, q -concavity, p -absolutely summing operators, etc. Examples. Two diagrams summarizing the various connections.	
g. The Approximation Property and Banach Lattices	102
Examples of Banach lattices and of subspaces of l_p , $p \neq 2$, without the B.A.P. The connection between the type and the cotype of a space and the existence of subspaces without the B.A.P. A space different from l_2 all of whose subspaces have the B.A.P.	
2. Rearrangement Invariant Function Spaces	114
a. Basic Definitions, Examples and Results	114
The definition of r.i. function spaces on $[0,1]$ and $[0,\infty)$. Conditional expectations. Interpolation between L_1 and L_∞ .	

b. The Boyd Indices	129
The definition of Boyd indices. Duality. The Rademacher functions in an r.i. function space on $[0, 1]$. The characterization of Boyd indices in terms of existence of l_p^n 's for all n on disjoint vectors having the same distribution. Operators of weak type (p, q) . Boyd's interpolation theorem.	
c. The Haar and the Trigonometric Systems	150
Basic results on martingales. The unconditionality of the Haar system in $L_p(0, 1)$ spaces ($1 < p < \infty$) and in more general r.i. function spaces on $[0, 1]$. Reproducibility of bases. The boundedness of the Riesz projection and applications.	
d. Some Results on Complemented Subspaces	168
The isomorphism between an r.i. function space X on $[0, 1]$ with non-trivial Boyd indices and the spaces $X(l_2)$ and $\text{Rad } X$. Complemented subspaces of X with an unconditional basis. Subspaces spanned by a subsequence of the Haar system. R.i. spaces with non-trivial Boyd indices are primary.	
e. Isomorphisms Between r.i. Function Spaces; Uniqueness of the r.i. Structure	181
Isomorphic embeddings. Classification of symmetric basic sequences in r.i. function spaces of type 2. Uniqueness of the r.i. structure for $L_p(0, 1)$ spaces and for $(q < 2)$ -concave r.i. spaces. Other applications.	
f. Applications of the Poisson Process to r.i. Function Spaces	202
The isomorphism between $L_p(0, 1)$ and $L_p(0, \infty) \cap L_2(0, \infty)$ for $p > 2$. R.i. function spaces in $[0, \infty)$ isomorphic to a given r.i. function space on $[0, 1]$. Isometric embeddings of $L_r(0, 1)$ into $L_p(0, 1)$ for $1 \leq p < r < 2$ and in other r.i. function spaces. The complementation of the spaces $X_{p, 2}$ in $L_p(0, 1)$, $p > 2$ and generalizations.	
g. Interpolation Spaces and their Applications	215
Interpolation pairs. General interpolation spaces and applications to the construction of r.i. function spaces without unique r.i. structure. The Lions-Peetre method of interpolation. Uniform convexity and type in interpolation spaces.	
References	233
Subject Index	239