

Joram Lindenstrauss Lior Tzafriri

Classical Banach Spaces I

Sequence Spaces

Reprint of the 1977 Edition

Classical Banach Spaces II

Function Spaces

Reprint of the 1979 Edition



Springer

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