

Series on Concrete and Applicable Mathematics – Vol.10

# **THEORY AND EXAMPLES OF ORDINARY DIFFERENTIAL EQUATIONS**

**Chin-Yuan Lin**

National Central University, Taiwan

*Published by*

World Scientific Publishing Co. Pte. Ltd.

5 Toh Tuck Link, Singapore 596224

USA office: 27 Warren Street, Suite 401-402, Hackensack, NJ 07601

UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

**British Library Cataloguing-in-Publication Data**

A catalogue record for this book is available from the British Library.

**THEORY AND EXAMPLES OF ORDINARY DIFFERENTIAL EQUATIONS**

**Series on Concrete and Applicable Mathematics — Vol. 10**

Copyright © 2011 by World Scientific Publishing Co. Pte. Ltd.

*All rights reserved. This book, or parts thereof, may not be reproduced in any form or by any means, electronic or mechanical, including photocopying, recording or any information storage and retrieval system now known or to be invented, without written permission from the Publisher.*

For photocopying of material in this volume, please pay a copying fee through the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923, USA. In this case permission to photocopy is not required from the publisher.

ISBN-13 978-981-4307-12-3

ISBN-10 981-4307-12-2

Printed in Singapore.

# Contents

<i>Preface</i>	vii
<i>List of Figures</i>	xiii
Chapter 1. Linear Equations	1
1. Introduction	1
2. Main Results	3
3. Examples	10
4. Proof of the Main Results	14
4.1. The Wronskian $W(x)$	14
4.2. Fundamental Sets of Solutions	16
4.3. Homogeneous Equations	22
4.4. Nonhomogeneous Equations	23
4.5. A Particular Solution	24
5. Equations of Special Type	31
5.1. First Order Equations	31
5.2. Euler Equations	39
5.3. Exact Equations	47
6. The Technique of the Laplace Transform	54
7. Problems	61
8. Solutions	63
Chapter 2. Systems of Linear First Order Equations	69
1. Introduction	69
2. Main Results	73
3. Examples	84
4. Proof of the Main Results	102
4.1. The Exponential Function $\exp(tA)$	102
4.2. The Wronskian $W(t)$	106
4.3. Methods of Computing the Fundamental Matrix $\exp(tA)$	109
4.4. Fundamental Sets of Solutions	115
4.5. Homogeneous Systems	115
4.6. Nonhomogeneous Systems	117
4.7. A Particular Solution	118
5. An Additional Method for Computing a Fundamental Matrix	127
6. Problems	137
7. Solutions	139
Chapter 3. Power Series Solutions	155
1. Introduction	155

2. Main Results	158
3. Examples	165
4. Proof of the Main Results	194
5. The Extension of the Main Results	204
6. Problems	221
7. Solutions	223
 Chapter 4. Adjoint Operators and Nonhomogeneous Boundary Value Problems	 239
1. Introduction	239
2. A Necessary and Sufficient Condition for Solvability	243
3. Examples	248
4. Complementary Properties between Adjoint Problems and Original Problems	261
5. An Abstract Result	277
5.1. Linear Operators	278
5.2. Adjoint Operators	280
5.3. Solvability Condition	283
6. Problems	285
7. Solutions	286
 Chapter 5. Green Functions	 311
1. Introduction	311
2. Main Results	313
3. Examples	318
4. Proof of the Main Results	328
5. More Theoretical Material	342
5.1. Riemann Integrals	342
5.2. Lebesgue Integrals	343
5.3. The Meaning of the Dirac Delta Function	347
6. Problems	352
7. Solutions	354
 Chapter 6. Eigenfunction Expansions	 363
1. Introduction	363
2. Main Results	367
3. Examples	368
4. Existence of Eigenvalues, and Completeness of Eigenfunctions	383
4.1. Properties of Eigenvalues	383
4.2. Properties of Eigenfunctions	386
4.3. Properties of Some Integral Operator	387
4.4. Existence of Eigenvalues and Eigenfunctions	390
4.5. Proof of the Main Results	399
5. Abstract Expansion Results	399
5.1. Linear Continuous Operators	400
5.2. Expansion Results for Compact Symmetric Operators	402
5.3. Properties of Compact Symmetric Operators	403
5.4. Proof of the Abstract Expansion Results	406
5.5. Examples from Ordinary Differential Operators	409

6. Problems	415
7. Solutions	416
Chapter 7. Long Time Behavior of Systems of Differential Equations	425
1. Introduction	425
2. Main Results for Linear Systems	427
3. Main Results for Nonlinear Systems	432
4. Examples for Linear Systems	437
5. Examples for Nonlinear Systems	446
6. Proof of the Main Results for Linear Systems	454
7. Proof of the Main Results for Nonlinear Systems	462
8. Problems	476
9. Solutions	482
Chapter 8. Existence and Uniqueness Theorems	499
1. Introduction	499
2. The Main Results with Continuous Coefficients	502
3. Proof of the Main Results	506
3.1. The Linear Case with Continuous Coefficients	506
3.2. The Nonlinear Case with Continuous Coefficients and with Uniqueness Result	513
3.3. The Nonlinear Case with Continuous Coefficients but without Uniqueness Result	517
4. The Results with Riemann or Lebesgue Integrable Coefficients	521
4.1. Existence and Uniqueness Theorems	522
4.2. Dependence on Initial Conditions and Parameters	532
<i>Bibliography</i>	537
<i>Index</i>	539