Thomas M. Liggett

Stochastic Interacting Systems: Contact, Voter and Exclusion Processes

With 6 Figures

Springer

Thomas M. Liggett
Mathematics Department
University of California
Los Angeles, CA 90095-1555
USA
email: tml@math.ucla.edu

Cataloging-in-Publication Data applied for

Die Deutsche Bibliothek – CIP-Einheitsaufnahme
Liggett, Thomas M.: Stochastic interacting systems: contact, voter and exclusion processes / Thomas M. Liggett. – Berlin; Heidelberg; New York; Barcelona; Hong Kong; London; Milan; Paris; Singapore; Tokyo: Springer 1999
(Grundlehren der mathematischen Wissenschaften; 324)

Mathematics Subject Classification (1991): 60K35

ISSN 0072-7830

ISBN 978-3-642-08529-1 ISBN 978-3-662-03990-8 (eBook)
DOI 10.1007/978-3-662-03990-8

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilm or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer-Verlag. Violations are liable for prosecution under the German Copyright Law.

© Springer-Verlag Berlin Heidelberg 1999

Originally published by Springer-Verlag Berlin Heidelberg New York in 1999.
Softcover reprint of the hardcover 1st edition 1999
Cover design: MetaDesign plus GmbH, Berlin
Typesetting: Photocomposed from the author's AMSTEX files after editing and reformatting by Kurt Mattes, Heidelberg, using a Springer TEX macro-package
Cover design: de'blik, Berlin
SPIN: 10728278 41/3143-5 4 3 2 1 0 Printed on acid-free paper

Contents

Background and Tools 1

The Processes 1
Invariant Measures 4
Reversible Measures 5
Coupling, Monotonicity and Attractiveness 6
Correlation Inequalities 8
Duality 11
Subadditivity 12
Oriented Percolation 13
Domination by Product Measures 14
Renewal Sequences and Logconvexity 16
Translation Invariant Measures 21
Some Ergodic Theory 22
Branching Processes 25
Some Queuing Theory 26
The Martingale CLT 29

Part I. Contact Processes 31

1. Preliminaries 31
Description of the Process 31
The Graphical Representation; Additivity 32
The Upper Invariant Measure 34
Duality 35
Convergence 36
Monotonicity and Continuity in λ 38
Rate of Growth 40
Survival and Extinction; Critical Values 42
Preview of Part I 44

2. The Process on the Integer Lattice Z^d 44
The Boundary of a Big Box Has Many Infected Sites 45
The Finite Space-Time Condition 50
Comparison with Oriented Percolation 51
First Consequences of the Percolation Comparison 54

Exponential Bounds in the Supercritical Case 57
Exponential Decay Rates in the Subcritical Case 60
A Critical Exponent Inequality 69

3. The Process on $\{1, \ldots, N\}^d$ 71
The Subcritical Case 72
The Supercritical Case 74

4. The Process on the Homogeneous Tree T_d 78
Some Critical Value Bounds 79
Branching Random Walk 80
Back to the Contact Process – the Function ϕ 86
Extinction at the First Critical Value 91
Existence of an Intermediate Phase 94
The Sequence u and its Growth Parameter $\beta(\lambda)$ 96
The Complete Convergence Theorem 103
Continuity of the Survival Probability 104
The Growth Profile 105
Invariant Measures in the Intermediate Regime – First Construction . 109
Invariant Measures in the Intermediate Regime – Second Construction 119
Strict Monotonicity of $\beta(\lambda)$ 123

5. Notes and References 125

Part II. Voter Models 139

1. Preliminaries 139
Description of the Process 139
Clustering and Coexistence 140
The Linear Voter Model 140
The Threshold Voter Model 142
The Graphical Representation 142
Duality when $T = 1$ 143
Preview of Part II 145

2. Models with General Threshold and Range 146
Fixation for Large Thresholds 146
Clustering in One Dimension 147
Coexistence; the Threshold Contact Process 151
The Threshold Contact Process with Large Range 153
The Threshold Voter Model with Large Range 155

3. Models with Threshold = 1 155
Duality for the Threshold Contact Process, $T = 1$ 156
Reduction to One Dimension 158
The Convolution Equation 159
The Density 162

The Renewal Sequence 167
Existence of a Nontrivial Invariant Measure 174
Nonnegativity for Sets that Contain No Singletons 180
Nonnegativity for General Sets 184
Strings of Length One 185
Strings of Length Greater than One 191

4. Notes and References 201

Part III. Exclusion Processes 209

1. Preliminaries 209
Description of the Process 209
Invariant Measures 210
Symmetric Systems 212
Coupling; the Graphical Representation 215
Translation Invariant Systems 215
First and Second Class Particles 218
The Tagged Particle Process 219
Preview of Part III 220

2. Asymmetric Processes on the Integers 220
Heuristics 222
Basic Assumption; Expected Results 224
Location of the Shock 225
Another View of the Shock 226
An Invariant Measure for the Process Viewed from X_t 230
The Process X_t Identifies the Shock 232
The Process Z_t also Identifies the Shock 234
Behavior of the Shock – First Moments 238
Behavior of the Shock – Weak Law of Large Numbers 240
Behavior of the Shock – Second Moments 242
Central Limit Behavior of the Shock 253
Dynamic Phase Transition 258

3. Invariant Measures for Processes on $\{1, \ldots, N\}$ 261
The Matrix Approach 262
Properties of the Matrices 264
Examples of Matrices D and E 266
Correlation Functions 268
The Partition Function 269
The Current 272
The Limiting Measure 273
An Application – the Process with a Blockage 276

4. The Tagged Particle Process 278
The Process Viewed from the Tagged Particle; First Decomposition . 278

Invariance and Ergodicity of the Environment 280
The Law of Large Numbers for X_t 284
Asymptotic Normality for M_t 285
The Second Decomposition – Beginning 286
The Basic Assumption 288
The Second Decomposition – Conclusion 290
Asymptotic Normality for X_t 294
The Limit Is Not Degenerate 295

5. Notes and References 298

Bibliography 317

Index 331