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Conical Refraction and Higher Microlocalization

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Preface

These notes focus on some results concerning propagation of analytic microlocal singularities for solutions of partial differential equations with characteristics of variable multiplicity, and on the tools from the theory of higher involutive microlocalization needed in the proofs.

The simplest model to which the results apply is Maxwell's system for homogeneous anisotropic optical media (typical examples of which are crystals); then the underlying physical phenomenon is that of conical refraction.

The main difficulty in the study of operators with characteristics of variable multiplicity stems from the fact that the characteristic variety of such operators is not smooth. Indeed, near a singular point, a number of constructions usually performed in the study of the propagation of singularities will degenerate or break down. In the analytic category, these difficulties can be best investigated from the point of view of higher microlocalization.

Unfortunately, none of the theories on higher analytic microlocalization in use nowadays completely covers the situation that we encounter later in the notes. Rather than adapting or extending the existing theories to the present needs, we have chosen to build up a new theory from a uniform point of view. Actually the results on higher microlocalization are sufficiently well delimited from the other results of the text and could in principle be read independently of the rest. Special emphasis is put, on the other hand, upon the relation and interplay between the results on propagation of microlocal singularities and similar results and constructions in geometrical optics.

All microlocalization processes seem to follow some underlying common pattern. Therefore some overlap with other articles and books on higher microlocalization has been inevitable. It is also clear in this situation that we have been greatly influenced by the published literature. However, the point of view on higher microlocalization taken here is different from both that of Kashiwara-Laurent on second microlocalization, and from that of Sjöstrand-Lebeau on higher microlocalization. In particular, the intersection with the Astérisque volume of Sjöstrand and the Birkhäuser volume of Laurent (see references) is reasonably small. Otherwise, the text is based to a large extent on results that were obtained by the author in the last few years and have not been published in detail before.

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