

P G Lemarié-Rieusset

Recent developments in the Navier-Stokes problem

Library of Congress Cataloging-in-Publication Data

Lemarié, Pierre Gilles, 1960-

Recent developments in the Navier-Stokes problem / P.G. Lemarié-Rieusset.

p. cm. -- (Chapman & Hall/CRC research notes in mathematics series ; 431)

Includes bibliographical references and index.

ISBN 1-58488-220-4 (alk. paper)

I. Navier-Stokes equations. I. Title. II. Series.

QA374 .L39 2002

515'.353--dc21

2002018858

This book contains information obtained from authentic and highly regarded sources. Reprinted material is quoted with permission, and sources are indicated. A wide variety of references are listed. Reasonable efforts have been made to publish reliable data and information, but the author and the publisher cannot assume responsibility for the validity of all materials or for the consequences of their use.

Neither this book nor any part may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopying, microfilming, and recording, or by any information storage or retrieval system, without prior permission in writing from the publisher.

The consent of CRC Press LLC does not extend to copying for general distribution, for promotion, for creating new works, or for resale. Specific permission must be obtained in writing from CRC Press LLC for such copying.

Direct all inquiries to CRC Press LLC, 2000 N.W. Corporate Blvd., Boca Raton, Florida 33431.

Trademark Notice: Product or corporate names may be trademarks or registered trademarks, and are used only for identification and explanation, without intent to infringe.

Visit the CRC Press Web site at www.crcpress.com

© 2002 by Chapman & Hall/CRC

No claim to original U.S. Government works

International Standard Book Number 1-58488-220-4

Library of Congress Card Number 2002018858

Printed in the United States of America 2 3 4 5 6 7 8 9 0

Printed on acid-free paper

Table of contents

Introduction	1
Chapter 1: What is this book about?	3
Uniform weak solutions for the Navier–Stokes equations	5
Mild solutions	6
Energy inequalities	10
Part 1: Some results of real harmonic analysis	13
Chapter 2: Real interpolation, Lorentz spaces and Sobolev embeddings	15
A primer to real interpolation theory	15
Lorentz spaces	18
Sobolev inequalities	20
Chapter 3: Besov spaces and Littlewood–Paley decomposition	23
The Littlewood-Paley decomposition of tempered distributions	23
Besov spaces as real interpolation spaces of potential spaces	25
Homogeneous Besov spaces	28
Chapter 4: Shift-invariant Banach spaces of distributions and related Besov spaces	31
Shift-invariant Banach spaces of distributions	31
Besov spaces	34
Homogeneous spaces	35
Chapter 5: Vector-valued integrals	39
The case of Lebesgue spaces	39
Spaces $L^p(E)$	41
Heat kernel and Besov spaces	44
Chapter 6: Complex interpolation, Hardy space and Calderón–Zygmund operators	47
The Marcinkiewicz interpolation theorem and the Hardy–Littlewood maximal function	47
The complex method in interpolation theory	50
Atomic Hardy space and Calderón–Zygmund operators	51
Chapter 7: Vector-valued singular integrals	57
Calderón–Zygmund operators	57
Littlewood–Paley decomposition in L^p	62
Maximal $L^p(L^q)$ regularity for the heat kernel	64

Chapter 8: A primer to wavelets	67
Multiresolution analysis	68
Daubechies wavelets.	73
Multivariate wavelets	77
Chapter 9: Wavelets and functional spaces	79
Lebesgue spaces	79
Besov spaces	81
Singular integrals	88
Chapter 10: The space BMO	91
Carleson measures and the duality between \mathcal{H}^1 and BMO	91
The $T(1)$ theorem	95
The local Hardy space h^1 and the local space bmo	100
Part 2: A general framework for shift-invariant estimates for the Navier–Stokes equations	103
Chapter 11: Weak solutions for the Navier–Stokes equations	105
The Leray projection operator and the Oseen kernel	105
Elimination of the pressure	107
Differential formulation and the integral formulation for the Navier–Stokes equations	112
Chapter 12: Divergence-free vector wavelets	115
A short survey in divergence-free vector wavelets.	115
Bi-orthogonal bases	116
The div-curl theorem	120
Chapter 13: The mollified Navier–Stokes equations	123
The mollified equations	123
The limiting process	128
Mild solutions	130
Part 3: Classical existence results for the Navier–Stokes equations	133
Chapter 14: The Leray solutions for the Navier–Stokes equations	135
The energy inequality	135
Energy equality	139
Uniqueness theorems	142

Chapter 15: The Kato theory of mild solutions for the Navier–Stokes equations	145
Picard’s contraction principle	145
Kato’s mild solutions in H^s , $s \geq d/2 - 1$	148
Kato’s mild solutions in L^p , $p \geq d$	151
Part 4: New approaches to mild solutions	157
Chapter 16: The mild solutions of Koch and Tataru	159
The space BMO^{-1}	159
Local and global existence of solutions	162
Fourier transform, Navier–Stokes and $BMO^{(-1)}$	167
Chapter 17: Generalization of the L^p theory: Navier–Stokes and local measures	171
Shift-invariant spaces of local measures	171
Kato’s theorem for local measures: the direct approach	173
Kato’s theorem for local measures: the role of $B_\infty^{-1,\infty}$	175
Chapter 18: Further results for local measures	179
The role of the Morrey–Campanato space $M^{1,d}$ and of $bmo^{(-1)}$	179
A persistency theorem	181
Some alternate proofs for the existence of global solutions	183
Chapter 19: Regular initial values	189
Cannone’s adapted spaces	189
Sobolev spaces and Besov spaces of positive order	192
Persistency results	194
Chapter 20: Besov spaces of negative order	197
$L^p(L^q)$ solutions	197
Potential spaces and Besov spaces	200
Persistency results	202
Chapter 21: Pointwise multipliers of negative order	205
Multipliers and Morrey–Campanato spaces	205
Solutions in X_r	211
Perturbated Navier–Stokes equations	215
Chapter 22: Further adapted spaces for the Navier–Stokes equations	221
The analysis of Meyer and Muschietti	221
The case of Besov spaces of null regularity	226
The analysis of Auscher and Tchamitchian	226
Chapter 23: Cannone’s approach of self-similarity	233
Besov spaces	233
The Lorentz space $L^{d,\infty}$	239
Asymptotic self-similarity	241

**Part 5: Decay and regularity results for weak
and mild solutions**

245

Chapter 24: Solutions of the Navier–Stokes equations are space-analytical	247
The Le Jan and Sznitman solutions	247
Analyticity of solutions in $\dot{H}^{d/2-1}$	249
Analyticity of solutions in L^d	250
Chapter 25: Space localization and Navier–Stokes equations	255
The molecules of Furioli and Terraneo	255
Spatial decay of velocities	260
Vorticities are well localized	264
Chapter 26: Time decay for the solutions to the Navier–Stokes equations	267
Wiegner's fundamental lemma and Schonbek's Fourier splitting device	267
Decay rates for the L^2 norm	268
Optimal decay rate for the L^2 norm	272
Chapter 27: Uniqueness of L^d solutions	277
The uniqueness problem.	277
Uniqueness in L^d	279
The case of Morrey–Campanato spaces	285
Chapter 28: Further results on uniqueness of mild solutions	289
Nonboundedness of the bilinear operator B on $\mathcal{C}([0, T], (L^d)^d)$	289
Uniqueness in $L^\infty(L^d)$ ($d \geq 4$)	291
A uniqueness result in $\dot{B}_\infty^{-1, \infty}$	293
Chapter 29: Stability and Lyapunov functionals	303
Stability in Lebesgue norms	303
A new Bernstein inequality	308
Stability and Besov norms	309

**Part 6: Local energy inequalities for the
Navier–Stokes equations on \mathbb{R}^3**

315

Chapter 30: The Caffarelli, Kohn, and Nirenberg regularity criterion	317
Suitable solutions	317
A fundamental inequality	322
The regularity criterion	324
Chapter 31: On the dimension of the set of singular points	331
Singular times	331
Hausdorff dimension of the set of singularities for a suitable solution ..	332
The second regularity criterion of Caffarelli, Kohn, and Nirenberg	

Chapter 32: Local existence (in time) of suitable local square-integrable weak solutions	341
Size estimates for \vec{u}_ϵ	342
Local existence of solutions	346
Decay estimates for suitable solutions	348
Chapter 33: Global existence of suitable local square-integrable weak solutions	353
Regularity of uniformly locally L^2 suitable solutions	353
A generalized Von Wahl uniqueness theorem	354
Global existence of uniformly locally L^2 suitable solutions	360
Chapter 34: Leray's conjecture on self-similar singularities	363
Hopf's strong maximum principle	363
The C_0 self-similar Leray solutions are equal to 0	364
The case of local control	367
Conclusion	373
Chapter 35: Singular initial values	375
Allowed initial values	375
Maximal regularity and critical spaces.	376
Mixed initial values	377
References	381
Bibliography	383
Author index	391
Subject index	393