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Alexander Leitsch

The Resolution Calculus

With 36 Figures



Springer

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Preface

The History of the Book

In August 1992 the author had the opportunity to give a course on resolution theorem proving at the Summer School for Logic, Language, and Information in Essex. The challenge of this course (a total of five two-hour lectures) consisted in the selection of the topics to be presented. Clearly the first selection has already been made by calling the course “resolution theorem proving” instead of “automated deduction”. In the latter discipline a remarkable body of knowledge has been created during the last 35 years, which hardly can be presented exhaustively, deeply and uniformly at the same time. In this situation one has to make a choice between a survey and a detailed presentation with a more limited scope. The author decided for the second alternative, but does not suggest that the other is less valuable. Today resolution is only one among several calculi in computational logic and automated reasoning. However, this does not imply that resolution is no longer up to date or its potential exhausted. Indeed the loss of the “monopoly” is compensated by new applications and new points of view. It was the purpose of the course mentioned above to present such new developments of resolution theory. Thus besides the traditional topics of completeness of refinements and redundancy, aspects of termination (resolution decision procedures) and of complexity are treated on an equal basis. The script of the course on resolution theorem proving appeared (in an improved version) as AILA preprint [Lei93a] and represents the skeleton of this book.

How to Use this Book

Chapters 2 (basics of resolution), 3 (refinements), and 4 (deletion) can be used for a traditional graduate course on resolution. However, some results and methods already point to the more advanced Chapters 5 (resolution decision procedures) and 6 (complexity). All the methods and results needed in Chapters 5 and 6 are provided in the first part of the book. Therefore, in principle, every student familiar with the basics of first-order logic can read the whole book. There are a few points where a basic knowledge of the theory of computability is needed to (fully) understand the material. On the other hand, Chapters 5 and 6 contain results of recent research, most of it published during the last five years. Thus the book is also addressed to

researchers working in the field of automated deduction; to them it should offer a source of information about new trends and paradigms in the form of a unified and systematic presentation.

Acknowledgments

First of all my thanks go to Daniele Mundici. It was he who invited me to give a course on resolution theorem proving at the Summer School LLI in Essex; later he encouraged me to transform the course script into a AILA preprint, which improved the substance of the material. Together with Cinzia Ferrari (my thanks to her too) he also read and checked a draft of this book and suggested many improvements. Reiner Hähnle thoroughly checked the first four chapters, found many errors and proposed several substantial changes and improvements. I also thank him for using the preprint mentioned above as basis of a course on automated theorem proving; his experience and observations made on this occasion influenced the book already in an early stage. Christian Fermüller read a draft of the whole book and gave many valuable hints to improve the presentation. Robert Matzinger not only checked the draft, he somehow traversed the material to its utmost depth. He spent several months on this task and his contribution to the improvement of the book cannot be overestimated. Reinhard Pichler read most of the book and made an interesting observation in Chapter 3. Matthias Baaz gave me some very valuable comments on Chapter 6 (which is based almost entirely on our common research). Special thanks go to the referees who wrote very competent and valuable reports on this book; their comments led to essential improvements of form and content. Gernot Salzer strongly assisted in \LaTeX problems and substantially contributed to the physical existence of the text. I am very grateful to Franziska Gusel for typing two thirds of the manuscript – an activity extending her usual work and inflicting additional stress. Many thanks go to Springer-Verlag, in particular to Hans Wössner and J. Andrew Ross, for their assistance and guidance in the difficult matter of transforming a text into a real book. Last, but not least, I have to thank my wife Marjan and my two sons David and Emanuel for their patience and tolerance during my work on this book. During the last four years I gave more to “resolution” than to them – sometimes forgetting that we are humans rather than scientific machines.

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