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978-0-521-77968-5 - Integral: An Easy Approach after Kurzweil and Henstock

Lee Peng Yee and Rudolf Vyborny

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