

# Functional Analysis

Applications in Mechanics and Inverse Problems

2<sup>nd</sup> Edition

by

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