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1. *John:* Partial Differential Equations, 4th ed.
2. *Sirovich:* Techniques of Asymptotic Analysis.
3. *Hale:* Theory of Functional Differential Equations, 2nd ed.
4. *Percus:* Combinatorial Methods.
5. *von Mises/Friedrichs:* Fluid Dynamics.
6. *Freiberger/Grenander:* A Short Course in Computational Probability and Statistics.
7. *Pipkin:* Lectures on Viscoelasticity Theory.
8. *Giacaglia:* Perturbation Methods in Non-linear Systems.
9. *Friedrichs:* Spectral Theory of Operators in Hilbert Space.
10. *Stroud:* Numerical Quadrature and Solution of Ordinary Differential Equations.
11. *Wolovich:* Linear Multivariable Systems.
12. *Berkovitz:* Optimal Control Theory.
13. *Bluman/Cole:* Similarity Methods for Differential Equations.
14. *Yoshizawa:* Stability Theory and the Existence of Periodic Solution and Almost Periodic Solutions.
15. *Braun:* Differential Equations and Their Applications, 3rd ed.
16. *Lefschetz:* Applications of Algebraic Topology.
17. *Collatz/Wetterling:* Optimization Problems.
18. *Grenander:* Pattern Synthesis: Lectures in Pattern Theory, Vol. I.
19. *Marsden/McCracken:* Hopf Bifurcation and Its Applications.
20. *Driver:* Ordinary and Delay Differential Equations.
21. *Courant/Friedrichs:* Supersonic Flow and Shock Waves.
22. *Rouche/Habets/Laloy:* Stability Theory by Liapunov's Direct Method.
23. *Lamperti:* Stochastic Processes: A Survey of the Mathematical Theory.
24. *Grenander:* Pattern Analysis: Lectures in Pattern Theory, Vol. II.
25. *Davies:* Integral Transforms and Their Applications, 2nd ed.
26. *Kushner/Clark:* Stochastic Approximation Methods for Constrained and Unconstrained Systems.
27. *de Boor:* A Practical Guide to Splines.
28. *Keilson:* Markov Chain Models—Rarity and Exponentiality.
29. *de Veubeke:* A Course in Elasticity.
30. *• niatycki:* Geometric Quantization and Quantum Mechanics.
31. *Reid:* Sturmian Theory for Ordinary Differential Equations.
32. *Meis/Markowitz:* Numerical Solution of Partial Differential Equations.
33. *Grenander:* Regular Structures: Lectures in Pattern Theory, Vol. III.
34. *Kevorkian/Cole:* Perturbation Methods in Applied Mathematics.
35. *Carr:* Applications of Centre Manifold Theory.
36. *Bengtsson/Ghil/Källén:* Dynamic Meteorology: Data Assimilation Methods.
37. *Saperstone:* Semidynamical Systems in Infinite Dimensional Spaces.
38. *Lichtenberg/Lieberman:* Regular and Chaotic Dynamics, 2nd ed.
39. *Piccini/Stampacchia/Vidossich:* Ordinary Differential Equations in $\mathbf{R}^n$.
40. *Naylor/Sell:* Linear Operator Theory in Engineering and Science.
41. *Sparrow:* The Lorenz Equations: Bifurcations, Chaos, and Strange Attractors.
42. *Guckenheimer/Holmes:* Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields.
43. *Ockendon/Taylor:* Inviscid Fluid Flows.
44. *Pazy:* Semigroups of Linear Operators and Applications to Partial Differential Equations.
45. *Glashoff/Gustafson:* Linear Operations and Approximation: An Introduction to the Theoretical Analysis and Numerical Treatment of Semi-Infinite Programs.
46. *Wilcox:* Scattering Theory for Diffraction Gratings.
47. *Hale et al:* An Introduction to Infinite Dimensional Dynamical Systems—Geometric Theory.
48. *Murray:* Asymptotic Analysis.
49. *Ladyzhenskaya:* The Boundary-Value Problems of Mathematical Physics.
50. *Wilcox:* Sound Propagation in Stratified Fluids.
51. *Golubitsky/Schaeffer:* Bifurcation and Groups in Bifurcation Theory, Vol. I.
52. *Chipot:* Variational Inequalities and Flow in Porous Media.
53. *Majda:* Compressible Fluid Flow and System of Conservation Laws in Several Space Variables.
54. *Wasow:* Linear Turning Point Theory.
55. *Yosida:* Operational Calculus: A Theory of Hyperfunctions.
56. *Chang/Howes:* Nonlinear Singular Perturbation Phenomena: Theory and Applications.
57. *Reinhardt:* Analysis of Approximation Methods for Differential and Integral Equations.
58. *Dwoyer/Hussaini/Voigt (eds):* Theoretical Approaches to Turbulence.
59. *Sanders/Verhulst:* Averaging Methods in Nonlinear Dynamical Systems.
60. *Ghil/Childress:* Topics in Geophysical Dynamics: Atmospheric Dynamics, Dynamo Theory and Climate Dynamics.

(continued following index)

Andrzej Lasota Michael C. Mackey

Chaos, Fractals, and Noise

Stochastic Aspects of Dynamics

Second Edition

With 48 Illustrations

Springer

Andrzej Lasota
Institute of Mathematics
Silesian University
ul. Bankowa 14
Katowice 40-058, Poland

Michael C. Mackey
Center of Nonlinear Dynamics
McGill University
Montreal, Quebec H3G 1Y6
Canada

Editors

J.E. Marsden
Control and Dynamical Systems, 107-81
California Institute of Technology
Pasadena, CA 91125
USA

L. Sirovich
Division of Applied Mathematics
Brown University
Providence, RI 02912
USA

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To the memory of

Maria Ważewska-Czyżewska

Preface to the Second Edition

The first edition of this book was originally published in 1985 under the title "Probabilistic Properties of Deterministic Systems." In the intervening years, interest in so-called "chaotic" systems has continued unabated but with a more thoughtful and sober eye toward applications, as befits a maturing field. This interest in the serious usage of the concepts and techniques of nonlinear dynamics by applied scientists has probably been spurred more by the availability of inexpensive computers than by any other factor. Thus, computer experiments have been prominent, suggesting the wealth of phenomena that may be resident in nonlinear systems. In particular, they allow one to observe the interdependence between the deterministic and probabilistic properties of these systems such as the existence of invariant measures and densities, statistical stability and periodicity, the influence of stochastic perturbations, the formation of attractors, and many others. The aim of the book, and especially of this second edition, is to present recent theoretical methods which allow one to study these effects.

We have taken the opportunity in this second edition to not only correct the errors of the first edition, but also to add substantially new material in five sections and a new chapter. Thus, we have included the additional dynamic property of sweeping (Chapter 5) and included results useful in the study of semigroups generated by partial differential equations (Chapters 7 and 11) as well as adding a completely new Chapter 12 on the evolution of distributions. The material of this last chapter is closely related to the subject of iterated function systems and their attractors-fractals. In addition, we have added a set of exercises to increase the utility of the work for graduate courses and self-study.

In addition to those who helped with the first edition, we would like to thank K. Alligood (George Mason), P. Kamthan, J. Losson, I. Nechayeva, N. Provatas (McGill), and A. Longtin (Ottawa) for their comments.

A.L.
M.C.M.

Preface to the First Edition

This book is about densities. In the history of science, the concept of densities emerged only recently as attempts were made to provide unifying descriptions of phenomena that appeared to be statistical in nature. Thus, for example, the introduction of the Maxwellian velocity distribution rapidly led to a unification of dilute gas theory; quantum mechanics developed from attempts to justify Planck's ad hoc derivation of the equation for the density of blackbody radiation; and the field of human demography grew rapidly after the introduction of the Gompertzian age distribution.

From these and many other examples, as well as the formal development of probability and statistics, we have come to associate the appearance of densities with the description of large systems containing inherent elements of uncertainty. Viewed from this perspective one might find it surprising to pose the questions: "What is the smallest number of elements that a system must have, and how much uncertainty must exist, before a description in terms of densities becomes useful and/or necessary?" The answer is surprising, and runs counter to the intuition of many. A one-dimensional system containing only one object whose dynamics are completely deterministic (no uncertainty) can generate a density of states! This fact has only become apparent in the past half-century due to the pioneering work of E. Borel [1909], A. Rényi [1957], and S. Ulam and J. von Neumann. These results, however, are not generally known outside that small group of mathematicians working in ergodic theory.

The past few years have witnessed an explosive growth in interest in physical, biological, and economic systems that could be profitably studied using densities. Due to the general inaccessibility of the mathematical literature to the nonmathematician, there has been little diffusion of the concepts and techniques from ergodic theory into the study of these "chaotic" systems. This book attempts to bridge that gap.

Here we give a unified treatment of a variety of mathematical systems generating densities, ranging from one-dimensional discrete time transformations through continuous time systems described by integro-partial-differential equations. We have drawn examples from a variety of the sciences to illustrate the utility of the techniques we present. Although the range of these examples is not encyclopedic, we feel that the ideas presented here may prove useful in a number of the applied sciences.

This book was organized and written to be accessible to scientists with a knowledge of advanced calculus and differential equations. In various places, basic concepts from measure theory, ergodic theory, the geometry of manifolds, partial differential equations, probability theory and Markov processes, and stochastic integrals and differential equations are introduced. This material is presented only as needed, rather than as a discrete unit at the beginning of the book where we felt it would form an almost insurmountable hurdle to all but the most persistent. However, in spite of our presentation of all the necessary concepts, we have not attempted to offer a compendium of the existing mathematical literature.

The one mathematical technique that touches every area dealt with is the use of the lower-bound function (first introduced in Chapter 5) for proving the existence and uniqueness of densities evolving under the action of a variety of systems. This, we feel, offers some partial unification of results from different parts of applied ergodic theory.

The first time an important concept is presented, its name is given in bold type. The end of the proof of a theorem, corollary, or proposition is marked with a ■; the end of a remark or example is denoted by a □.

A number of organizations and individuals have materially contributed to the completion of this book.

In particular the National Academy of Sciences (U.S.A.), the Polish Academy of Sciences, the Natural Sciences and Engineering Research Council (Canada), and our home institutions, the Silesian University and McGill University, respectively, were especially helpful.

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Contents