

Applied Mathematical Sciences

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J.P. LaSalle

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FOREWORD

Professor J. P. LaSalle died on July 7, 1983 at the age of 67. The present book is being published posthumously with the careful assistance of Kenneth Meyer, one of the students of Professor LaSalle. It is appropriate that the last publication of Professor LaSalle should be on a subject which contains many interesting ideas, is very useful in applications and can be understood at an undergraduate level. In addition to making many significant contributions at the research level to differential equations and control theory, he was an excellent teacher and had the ability to make sophisticated concepts appear to be very elementary. Two examples of this are his books with N. Hasser and J. Sullivan on analysis published by Ginn and Co., 1949 and 1964, and the book with S. Lefschetz on stability by Liapunov's second method published by Academic Press, 1961. Thus, it is very fitting that the present volume could be completed.

Jack K. Hale
Kenneth R. Meyer

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