

Applied Mathematical Sciences

EDITORS

Fritz John

*Courant Institute of
Mathematical Sciences*
New York University
New York, NY 10012

J.E. Marsden

*Department of
Mathematics*
University of California
Berkeley, CA 94720

Lawrence Sirovich

*Division of
Applied Mathematics*
Brown University
Providence, RI 02912

ADVISORS

M. Ghil New York University

J.K. Hale Brown University

J. Keller Stanford University

K. Kirchgässner Universität Stuttgart

B. Matkowsky Northwestern University

J.T. Stuart Imperial College

A. Weinstein University of California

EDITORIAL STATEMENT

The mathematization of all sciences, the fading of traditional scientific boundaries, the impact of computer technology, the growing importance of mathematical-computer modelling and the necessity of scientific planning all create the need both in education and research for books that are introductory to and abreast of these developments.

The purpose of this series is to provide such books, suitable for the user of mathematics, the mathematician interested in applications, and the student scientist. In particular, this series will provide an outlet for material less formally presented and more anticipatory of needs than finished texts or monographs, yet of immediate interest because of the novelty of its treatment of an application or of mathematics being applied or lying close to applications.

The aim of the series is, through rapid publication in an attractive but inexpensive format, to make material of current interest widely accessible. This implies the absence of excessive generality and abstraction, and unrealistic idealization, but with quality of exposition as a goal.

Many of the books will originate out of and will stimulate the development of new undergraduate and graduate courses in the applications of mathematics. Some of the books will present introductions to new areas of research, new applications and act as signposts for new directions in the mathematical sciences. This series will often serve as an intermediate stage of the publication of material which, through exposure here, will be further developed and refined. These will appear in conventional format and in hard cover.

MANUSCRIPTS

The Editors welcome all inquiries regarding the submission of manuscripts for the series. Final preparation of all manuscripts will take place in the editorial offices of the series in the Division of Applied Mathematics, Brown University, Providence, Rhode Island.

Applied Mathematical Sciences

1. John: **Partial Differential Equations**, 4th ed.
2. Sirovich: **Techniques of Asymptotic Analysis**.
3. Hale: **Theory of Functional Differential Equations**, 2nd ed.
4. Percus: **Combinatorial Methods**.
5. von Mises/Friedrichs: **Fluid Dynamics**.
6. Freiberger/Grenander: **A Short Course in Computational Probability and Statistics**.
7. Pipkin: **Lectures on Viscoelasticity Theory**.
9. Friedrichs: **Spectral Theory of Operators in Hilbert Space**.
11. Wolovich: **Linear Multivariable Systems**.
12. Berkovitz: **Optimal Control Theory**.
13. Bluman/Cole: **Similarity Methods for Differential Equations**.
14. Yoshizawa: **Stability Theory and the Existence of Periodic Solutions and Almost Periodic Solutions**.
15. Braun: **Differential Equations and Their Applications**, 3rd ed.
16. Lefschetz: **Applications of Algebraic Topology**.
17. Collatz/Wetterling: **Optimization Problems**.
18. Grenander: **Pattern Synthesis: Lectures in Pattern Theory, Vol I**.
20. Driver: **Ordinary and Delay Differential Equations**.
21. Courant/Friedrichs: **Supersonic Flow and Shock Waves**.
22. Rouche/Habets/Laloy: **Stability Theory by Liapunov's Direct Method**.
23. Lamperti: **Stochastic Processes: A Survey of the Mathematical Theory**.
24. Grenander: **Pattern Analysis: Lectures in Pattern Theory, Vol. II**.
25. Davies: **Integral Transforms and Their Applications**, 2nd ed.
26. Kushner/Clark: **Stochastic Approximation Methods for Constrained and Unconstrained Systems**.
27. de Boor: **A Practical Guide to Splines**.
28. Keilson: **Markov Chain Models—Rarity and Exponentiality**.
29. de Veubeke: **A Course in Elasticity**.
30. Sniatycki: **Geometric Quantization and Quantum Mechanics**.
31. Reid: **Sturmian Theory for Ordinary Differential Equations**.
32. Meis/Markowitz: **Numerical Solution of Partial Differential Equations**.
33. Grenander: **Regular Structures: Lectures in Pattern Theory, Vol. III**.
34. Kevorkian/Cole: **Perturbation Methods in Applied Mathematics**.
35. Carr: **Applications of Centre Manifold Theory**.
36. Bengtsson/Ghil/Källén: **Dynamic Meteorology: Data Assimilation Methods**.
37. Saperstone: **Semidynamical Systems in Infinite Dimensional Spaces**.
38. Lichtenberg/Lieberman: **Regular and Stochastic Motion**.

(continued on inside back cover)

J.P. LaSalle

The Stability and Control of Discrete Processes

With 15 Illustrations



Springer Science+Business Media, LLC

AMS Subject Classifications: 34A30, 34D99, 34H05, 49E99, 58F10, 93C05, 93C55

Library of Congress Cataloging in Publication Data

LaSalle, Joseph P.

The stability and control of discrete processes.

(Applied mathematical sciences; v. 62)

Bibliography: p.

Includes index.

1. Control theory. 2. Stability. 3. Differential equations. I. Title. II. Series: Applied mathematical sciences (Springer-Verlag New York Inc.); 62.

QA1.A647 vol. 62 510 s 86-17757

[QA402.3] [629.8'312]

©1986 by Springer Science+Business Media New York

Originally published by Springer-Verlag New York Inc. in 1986

All rights reserved. No part of this book may be translated or reproduced in any form without written permission from Springer Science+Business Media, LLC.

9 8 7 6 5 4 3 2 1

ISBN 978-0-387-96411-9 ISBN 978-1-4612-1076-4 (eBook)

DOI 10.1007/978-1-4612-1076-4

FOREWORD

Professor J. P. LaSalle died on July 7, 1983 at the age of 67. The present book is being published posthumously with the careful assistance of Kenneth Meyer, one of the students of Professor LaSalle. It is appropriate that the last publication of Professor LaSalle should be on a subject which contains many interesting ideas, is very useful in applications and can be understood at an undergraduate level. In addition to making many significant contributions at the research level to differential equations and control theory, he was an excellent teacher and had the ability to make sophisticated concepts appear to be very elementary. Two examples of this are his books with N. Hasser and J. Sullivan on analysis published by Ginn and Co., 1949 and 1964, and the book with S. Lefschetz on stability by Liapunov's second method published by Academic Press, 1961. Thus, it is very fitting that the present volume could be completed.

Jack K. Hale
Kenneth R. Meyer

TABLE OF CONTENTS

	page
1. Introduction	1
2. Liapunov's direct method	7
3. Linear systems $x' = Ax$.	13
4. An algorithm for computing A^n .	19
5. A characterization of stable matrices. Computational criteria.	24
6. Liapunov's characterization of stable matrices. A Liapunov function for $x' = Ax$.	32
7. Stability by the linear approximation.	38
8. The general solution of $x' = Ax$. The Jordan Canonical Form.	40
9. Higher order equations. The general solution of $\psi(z)y = 0$.	45
10. Companion matrices. The equivalence of $x' = Ax$ and $\psi(z)y = 0$.	51
11. Another algorithm for computing A^n .	66
12. Nonhomogeneous linear systems $x' = Ax + f(n)$. Variation of parameters and undetermined coefficients.	70
13. Forced oscillations.	84
14. Systems of higher order equations $P(z)y = 0$. The equivalence of polynomial matrices.	87
15. The control of linear systems. Controllability.	94
16. Stabilization by linear feedback. Pole assignment.	107
17. Minimum energy control. Minimal time-energy feedback control.	117
18. Observability. Observers. State estimation. Stabilization by dynamic feedback.	125
References	147
Index	149