

Ilya S. Molchanov

Limit Theorems for Unions of Random Closed Sets

Springer-Verlag

Berlin Heidelberg New York

London Paris Tokyo

Hong Kong Barcelona

Budapest

Author

Ilya S. Molchanov
Department of Mathematics
Kiev Technological Institute of the Food Industry
Vladimirskaia 68
252017 Kiev, Ukraine
and
FB Mathematik
TU Bergakademie Freiberg
Bernhard-v.-Cotta-Str. 2
D-09596 Freiberg, Germany

Mathematics Subject Classification (1991): 60-02, 60D05, 60G55, 60G70, 26E25, 28B20, 52A22

ISBN 3-540-57393-3 Springer-Verlag Berlin Heidelberg New York
ISBN 0-387-57393-3 Springer-Verlag New York Berlin Heidelberg

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, re-use of illustrations, recitation, broadcasting, reproduction on microfilms or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer-Verlag. Violations are liable for prosecution under the German Copyright Law.

© Springer-Verlag Berlin Heidelberg 1993
Printed in Germany

2146/3140-543210 - Printed on acid-free paper

Contents

1	Distributions of Random Closed Sets	1
1.1	The Space of Closed Sets.	1
1.2	Random Closed Sets and Capacity Functionals.	2
1.3	Convex Random Sets.	5
1.4	Weak Convergence of Random Closed Sets.	7
1.5	Set-Theoretic Operations and Measurability.	11
1.6	Regularly Varying Functions.	13
2	Survey on Stability of Random Sets and Limit Theorems for Minkowski Addition	15
2.1	Expectation of a Random Closed Set.	15
2.2	A Strong Law of Large Numbers for Minkowski Sums.	16
2.3	A Central Limit Theorem for Random Sets.	17
2.4	Generalized Expectations of Random Sets	18
3	Infinite Divisibility and Stability of Random Sets with respect to Unions	29
3.1	Union-Stable Random Sets.	29
3.2	Examples of Union-stable Random Closed Sets.	35
3.3	Convex-Stable Random Sets.	38
3.4	Generalizations and Remarks.	42
4	Limit Theorems for Normalized Unions of Random Closed Sets	45
4.1	Sufficient Conditions for the Weak Convergence of Unions of Random Sets	45
4.2	Necessary Conditions in the Limit Theorem for Unions	49
4.3	Limit Theorems for Normalized Convex Hulls.	51
4.4	Limit Theorems for Unions and Convex Hulls of Special Random Sets	54
4.5	Further Remarks and Open Problems.	64
5	Almost Sure Convergence of Unions of Random Closed Sets	67
5.1	Almost Sure Convergence of Random Closed Sets.	67
5.2	Regularly Varying Capacities.	69
5.3	A Strong Law of Large Numbers for Unions of Random Closed Sets	71
5.4	Almost sure Limits for Unions of Special Random Sets	77

**6 Multivalued Regularly Varying Functions and Their Application to
Limit Theorems for Unions of Random Sets 85**

6.1 Definition of Multivalued Regular Variation 85

6.2 The Inversion Theorem for Multivalued Regularly Varying Functions . 89

6.3 Integrals on Multivalued Regularly Varying Functions 94

6.4 Limit Theorems for Unions: Multivalued Functions Approach 96

7 Probability Metrics in the Space of Random Sets Distributions 101

7.1 Definitions of Probability Metrics. 101

7.2 Some Inequalities between Probability Metrics. 106

7.3 Ideal Metrics for Random Closed Sets. 110

7.4 Applications to Limit Theorems for Unions. 114

8 Applications of Limit Theorems 123

8.1 Simulation of Stable Random Sets. 123

8.2 Estimation of Tail Probabilities for Volumes of Random Samples . . . 129

8.3 Convergence of Random Sets Generated by Graphs of Random Functions 134

8.4 Convergence of Random Processes Generated by Approximations of
Convex Compact Sets 139

8.5 A Limit Theorem for Intersections of Random Half-Spaces 142

References 147

Index 153