I. Kuzin
S. Pohozaev

Entire Solutions of Semilinear Elliptic Equations

Birkhäuser
Basel · Boston · Berlin

I. Kuzin and
S. Pohozaev
Steklov Mathematical Institute
42 Vavilova
117966 Moscow
Russia

1991 Mathematics Subject Classification 35J60, 35K57, 35Q55, 35Q40

A CIP catalogue record for this book is available from the Library of Congress,
Washington D.C., USA

Deutsche Bibliothek Cataloging-in-Publication Data
Kuzin, Il'ja A.:
Entire solutions of semilinear elliptic equations / I. Kuzin ;
S. Pohozaev. – Basel ; Boston ; Berlin : Birkhäuser, 1997
(Progress in nonlinear differential equations and their applications ;
Vol. 33)
ISBN-13: 978-3-0348-9962-8 e-ISBN-13: 978-3-0348-9250-6
DOI: 10.1007/978-3-0348-9250-6

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, re-use of illustrations, broadcasting, reproduction on microfilms or in other ways, and storage in data banks. For any kind of use whatsoever, permission of the copyright owner must be obtained.

© 1997 Birkhäuser Verlag, P.O. Box 133, CH-4010 Basel, Switzerland
Softcover reprint of the hardcover 1st edition 1997
Printed on acid-free paper produced from chlorine-free pulp. TCF ∞

9 8 7 6 5 4 3 2 1

Contents

Introduction 1

§ 0 Notation 4

Chapter 1
Classical Variational Method 5

§ 1 Preliminaries 5
§ 2 The Classical Method: Absolute Minimum 12
§ 3 Approximation by Bounded Domains 27
§ 4 Approximation for Problems on an Absolute Minimum 32
§ 5 The Monotonicity Method. Uniqueness of Solutions 36

Chapter 2
Variational Methods for Eigenvalue Problems 39

§ 6 Abstract Theorems 39
§ 7 The Equation $-\Delta u + a(\mathbf{x})|u|^{p-2}u - \lambda b(\mathbf{x})|u|^{q-2}u = 0$ 47
§ 8 Radial Solutions 55
§ 9 The Equation $-\Delta u + \lambda f(u) = 0$ 58
§ 10 The Equation $-\Delta u - \lambda|u|^{p-2}u - b(\mathbf{x})|u|^{q-2}u = 0$ 67
§ 11 The Comparison Method for Eigenvalue Problems (Concentration Compactness) 72
§ 12 Homogeneous Problems 81

Chapter 3
Special Variational Methods 83

§ 13 The Mountain Pass Method 83
§ 14 Behavior of PS-sequences. The Concentration Compactness (Comparison) Method 90
§ 15 A General Comparison Theorem. The Ground State. Examples for the Mountain Pass Method 106
§ 16 Behavior of PS-sequences in the Symmetric Case. Existence Theorems 113
§ 17 Nonradial Solutions of Radial Equations 120
§ 18 Methods of Bounded Domains Approximation 124

Chapter 4
Radial Solutions: The ODE Method 129

§ 19 Basic Techniques of the ODE Method 129
§ 20 Autonomous Equations in the N-dimensional Case 132
§ 21 Decaying Solutions. The One-dimensional Case 138
§ 22 The Phase Plane Method. The Emden-Fowler Equation 141
§ 23 Scaling 153
§ 24 Positive Solutions. The Shooting Method 154

Chapter 5
Other Methods 159

§ 25 The Method of Upper and Lower Solutions 159
§ 26 The Leray-Schauder Method 171
§ 27 The Method of A Priori Estimates 175
§ 28 The Fibering Method. Existence of Infinitely Many Solutions 205
§ 29 Nonexistence Results 217

Appendices 223

A Spaces and Functionals 223
B The Strauss Lemma 226
C Invariant Spaces 228
D The Schwarz Rearrangement 230
E The Mountain Pass Method 234

References 243
Index 249