

Contents

Introduction	xiii
Chapter 0. Preliminaries on valued and ordered modules	1
1. Valued modules	1
2. Valuation independence	5
3. Ordered modules	7
Chapter 1. Non-archimedean exponential fields	15
1. The natural valuation of an ordered field	15
2. The skeleton of $(K^{>0}, \cdot, 1, <)$	18
3. Formally exponential fields	22
4. Lexicographic (de)composition of exponentials	24
5. Exponentiation in power series fields	27
6. Extensions and maximality	29
7. The structure theory for countable exponential fields	31
Chapter 2. Valuation theoretic interpretation of the growth and Taylor axioms	33
1. The axiom schemes (GA) and (T)	33
2. (GA)-exponentials and the value group	34
3. Lifting exp from the residue field	36
4. (T)-exponentials on the infinitesimals	37
5. Conclusion	39
6. Countable exponential fields with growth properties	40
7. Natural contractions arising from logarithms	44
Chapter 3. The exponential rank	49
1. Convex valuations	49
2. The exponential analogue of the rank	52
3. (GA)- and (T_1) -prelogarithms	53
4. The shift map ζ_ℓ	56
5. Characterization of the exponential and the principal exponential rank	61
Chapter 4. Construction of exponential fields	65
1. w -Logarithmic cross-sections	65
2. A combinatorial result and its consequences	67
3. Existence of logarithmic cross-sections	70
4. From prelogarithms to logarithms	73
Chapter 5. Models for the elementary theory of the reals with restricted analytic functions and exponentiation	77
1. Twisting a group cross-section by an automorphism	77

2.	The exponential-logarithmic power series field	79
3.	Models of arbitrary principal exponential rank	83
Chapter 6. Exponential Hardy fields		89
1.	Some basic valuation theory	89
2.	Hardy fields	92
3.	Value groups	97
4.	The Hardy field of a polynomially bounded $+$ (exp) expansion	98
5.	Exponential boundedness	101
6.	Levels	102
7.	The Crucial Lemma for models of T_{an}	103
8.	Residue fields of \mathcal{F} -exp-log-closures	107
9.	A truncation free solution to the Hardy problem	111
10.	Undefinability of the Riemann ζ -function	113
Appendix A. The model theory of contraction groups		117
1.	Preliminaries	117
2.	Cuts in ordered Abelian groups	118
3.	Ordered abelian groups with contractions	120
4.	Weak o-minimality	136
Bibliography		155
Index		159
List of Notation		163