Learning and Geometry: Computational Approaches

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Table of Contents

Preface

The field of computational learning theory arose out of the desire to formally understand the process of learning. As potential applications to artificial intelligence became apparent, the new field grew rapidly. The learning of geometric objects became a natural area of study. The possibility of using learning techniques to compensate for unsolvability provided an attraction for individuals with an immediate need to solve such difficult problems.

Researchers at the Center for Night Vision were interested in solving the problem of interpreting data produced by a variety of sensors. Current vision techniques, which have a strong geometric component, can be used to extract features. However, these techniques fall short of useful recognition of the sensed objects. One potential solution is to incorporate learning techniques into the geometric manipulation of sensor data. As a first step toward realizing such a solution, the Systems Research Center at the University of Maryland, in conjunction with the Center for Night Vision, hosted a Workshop on Learning and Geometry in January of 1991. Scholars in both fields came together to learn about each others' field and to look for common ground, with the ultimate goal of providing a new model of learning from geometrical examples that would be useful in computer vision.

The papers in the volume are a partial record of that meeting. In addition to the research papers submitted for these proceedings we include the program for the workshop, a list of participants, and the introductory remarks concerning the purpose of the workshop prepared by Vincent Mirelli of the Center for Night Vision.

David Kueker
Carl Smith

List of Participants

John S. Baras
Eric B. Baum
Jürgen Bokowski
Roger Brockett
John Case
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Sanjeev R. Kulkarni
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Vincent Mirelli
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Kathleen Romanik
Haim Shvayster
Robert Sloan
Carl Smith
Vladmir Vapnik
Walter Whiteley
Rolf Wiehagen
Wu Wen-Tsun
Wlodek Zadrozny

Program for the Workshop

Tuesday, January 8, 1991

Plenary Talk, "Learning Patterns Based on Conditional Density Propagation," Roger Brockett, Harvard University

"Representing Geometric Configurations," Walter Whiteley, Champlain Regional College

"Mechanical Geometry Theorem Using Algbraic Methods," Shang-Ching Chou, University of Texas at Austin

Wednesday, January 9, 1991

Plenary Talk, "Some Aspects of Learning and Geometry in Computational Synthetic Geometry," Jurgen Bokowski, University of Darmstadt

"MDL Learning," Jorma Rissanen, IBM, Almaden Research Center

"Induction Principles in the Learning Theory," Vladimir Vapnik, Visitor, AT&T Bell Laboratories

Thursday, January 10, 1991

Plenary Talk, "Recursion Theoretic Learning Theory," John Case, University of Delaware

"Vague Predicates and Rules of Abduction," Wlodek Zadrozny, IBM Hawthorne

"'PAC' Learning from Noisy Data," Robert Sloan, University of Illinois at Chicago

Friday, January 11, 1991

Plenary Talk, "Geometry Theorem Proving in Euclidean, Descartesian, Hilbertian and Computerwise Fashions," Wen-Tsun Wu, Institute of System Sciences, China

"Naturalness in Inductive Inference," Rolf Wiehagen, der Humboldt Universität zu Berlin

"When Pluralistic Learners Behave Probabilistically for Finite Inductive Inference," Robert Daley, University of Pittsburgh

Introduction

Vincent Mirelli

Human vision can be interpreted as the function of learning the lowest complexity properties and invariants characterizing the useful partitions in classes of geometrical shapes (and of the geometrical components of complex figures). So it involves in an essential way geometry and learning. Thus, high-level human vision skills can be studied within the context of learning from geometrical examples. In turn, the learning process is related with the apparent ability of humans to establish and use a network of relationships between several different representations of classes of geometrical shapes at various levels classified as semantic or syntactic/symbolic/algebraic, depending on context.

Further progress in computer vision requires a careful reexamination of vision fundamentals along the interpretation described above, towards discovering the appropriate mathematical foundations and semantics which best fit vision problems. Towards this end we organized a cross disciplinary workshop at the Systems Research Center (now called the Institute for Systems Research) of the University of Maryland on "Learning and Geometry." The central objective of this workshop was to bring together several prominent mathematical researchers from fields which hold promise in establishing a solid mathematical framework linking learning and geometry in the context of vision. The topical fields represented in the conference are: computational linguistics, geometry theorem proving, synthetic, foundational and algebraic geometry. We provide below a brief description of the relevance and promise of each towards the central theme of this conference.

Computational learning [2], [11] addresses the problem of learning from examples. The Valiant model of learning [3] has been successful in several problems [4]. However, we believe that it needs to be extended and modified in order to address the specific human talents and skills involved in vision. The vision phenomenology is considerably more structured than previously addressed examples in the computational learning literature. Human exploits in a crucial way this additional structure and the interrelationships between model theoretic semantic structure and the mathematical representations in various formal systems. This type of interrelationships is analogous to the learning formation of natural language, where syntactical constraints are induced by the rules which map meanings into linear surfaces, and vice versa.

This interpretation makes it apparent that the field of *model-theoretic semantics* [5] is a natural context in which to formulate the understanding of these interrelationships. It is also appropriate for the organization of contributions from other topical fields represented in this conference. Model-theoretic semantics is at the heart of human learning, for instance in natural language learning. It is one of the goals of this conference to understand the role of model-theoretic semäntics in vision. In the context of learning and geometry (in particular as it relates to vision) we need methodologies that can efficiently link different semantic and syntactic representations and related models, which focus on different views of the empirical data and provide the framework for logical interference along different views (for instance, the local, continuous, global, discrete, algebraic, and symbolic aspects). In vision it is necessary to provide a framework for studying the linkages between these different representations and the ability to reason accross such semantic barriers. Model theoretic semantics and extentions can play that role.

The view expressed here for the mathematical foundations of vision is analogous to the view that has generated the modern field of *computational linguistics* in place of classical linguistics. From this point of view formal languages and *computational linguistics* [7] provide the mathematical framework for analyzing the structure of the various representations and the resulting syntactical constraints. In addition they provide the means for formal evaluation of different representations (e.g., expressive power of languages) and formal means for classifying languages and linking representations. Since learning ability (that is the concept of class of shapes being learnable) depends heavily on representation (i.e., the language used), and since we are interested in a framework that permits formulation of the concept of learnable based on concurrently linked representations, computational linguistics is seen as a significant methodological ingredient towards the overall theme of the conference.

The field of *geometry theorem proving* has been very successful in its goal [6], [9], while automatic theorem proving has not [8]. The reason is that automatic theorem proving tries to imitate the way human beings prove theorems, while geometry theorem proving is inspired by the relation between synthetic and analytic geometry. This relationship is analogous to that between syntax and semantics, which we believe is fundamental in the learning process. The additional relevance of this discipline on learning from geometrical example originates from the fact that there is a profound interpenetration among human talents of empirical learning, theorem proving, and calculus (i.e., chains of important algebraic operations performed almost without thinking). It is self evident, that this topical field, based on synthetic, foundational and algebraic geometry [1], [10], would also provide the formal framework to obtain the representations needed to characterize sets of geometrical shapes.

References

[1] *Fundamentals of Mathematics: Volume 2 Geometry*, H. Behnke, F. Bachman, K. Fladt, and H. Kunle, eds., MIT Press, Cambridge, MA, 1974.

[2] *Proceedings of the 1988 Workshop on Computational Learning Theory III*, Morgan Kaufman Publishers, 1988.

[3] L.G. Valiant, "A theory of the learnable," *Communications of the ACM*, **27**(11), Nov. 1984, pp. 1134–1142.

[4] H. Shvayster, "Learnable and nonlearnable visual concepts," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, **12**(5), May 1990, pp. 459–466.

[5] C.C. Chang and H.J. Keisler, *Model Theory*, North Holland, 1973.

[6] *Geometric Reasoning*, D. Kapur and J.L. Mundy, eds, MIT Press, Cambridge, MA, 1988.

[7] *Computation of Language: an Essay on Syntax, Semantics and Pragmatics in Natural Man-Machine Communication*, R. Hausser, Springer-Verlag, New York, 1989.

[8] *The Artificial Intelligence Debate: False Starts and Real Foundations*, S.R. Graubard, ed., MIT Press, Cambridge, MA, 1988.

[9] S.-C. Chou, Mechanical geometry theorem proving, *Mathematics and its Applications,* **41**, D. Reidel Publishing Co., 1988.

[10] B. Sturmfels and J. Bokowski, *Computational Synthetic Geometry*, Springer-Verlag, 1989.

[11] M.J. Kearns, *The Computational Complexity of Machine Learning*, MIT Press, Cambrige, MA, 1990.