

# Doing Bayesian Data Analysis

A Tutorial with R, JAGS,  
and Stan

EDITION

2

**JOHN K. KRUSCHKE**

Dept. of Psychological and Brain Sciences  
Indiana University, Bloomington



AMSTERDAM • BOSTON • HEIDELBERG • LONDON  
NEW YORK • OXFORD • PARIS • SAN DIEGO  
SAN FRANCISCO • SINGAPORE • SYDNEY • TOKYO

Academic Press is an imprint of Elsevier



# CONTENTS

<b>1. What's in This Book (Read This First!)</b> .....	<b>1</b>
1.1 Real people can read this book .....	1
1.2 What's in this book .....	3
1.3 What's new in the second edition? .....	6
1.4 Gimme feedback (be polite) .....	8
1.5 Thank you! .....	8
<b>PART I The Basics: Models, Probability, Bayes' Rule, and R</b> .....	<b>13</b>
<b>2. Introduction: Credibility, Models, and Parameters</b> .....	<b>15</b>
2.1 Bayesian inference is reallocation of credibility across possibilities .....	16
2.2 Possibilities are parameter values in descriptive models .....	22
2.3 The steps of bayesian data analysis .....	25
2.4 Exercises .....	31
<b>3. The R Programming Language</b> .....	<b>33</b>
3.1 Get the software .....	35
3.2 A simple example of R in action .....	36
3.3 Basic commands and operators in R .....	38
3.4 Variable types .....	42
3.5 Loading and saving data .....	53
3.6 Some utility functions .....	56
3.7 Programming in R .....	61
3.8 Graphical plots: opening and saving .....	69
3.9 Conclusion .....	69
3.10 Exercises .....	70
<b>4. What is This Stuff Called Probability?</b> .....	<b>71</b>
4.1 The set of all possible events .....	72
4.2 Probability: outside or inside the head .....	73
4.3 Probability distributions .....	78
4.4 Two-way distributions .....	89
4.5 Appendix: R code for figure 4.1 .....	93
4.6 Exercises .....	95

<b>5. Bayes' Rule</b> .....	<b>99</b>
5.1 Bayes' rule .....	100
5.2 Applied to parameters and data .....	105
5.3 Complete examples: estimating bias in a coin .....	108
5.4 Why Bayesian inference can be difficult .....	115
5.5 Appendix: R code for figures 5.1, 5.2, etc. ....	116
5.6 Exercises .....	118
<b>PART II All the Fundamentals Applied to Inferring a Binomial Probability</b> .....	<b>121</b>
<b>6. Inferring a Binomial Probability via Exact Mathematical Analysis</b> .....	<b>123</b>
6.1 The likelihood function: Bernoulli distribution .....	124
6.2 A description of credibilities: the beta distribution .....	126
6.3 The posterior beta .....	132
6.4 Examples .....	134
6.5 Summary .....	138
6.6 Appendix: R code for figure 6.4 .....	138
6.7 Exercises .....	139
<b>7. Markov Chain Monte Carlo</b> .....	<b>143</b>
7.1 Approximating a distribution with a large sample .....	145
7.2 A simple case of the Metropolis algorithm .....	146
7.3 The Metropolis algorithm more generally .....	156
7.4 Toward Gibbs sampling: estimating two coin biases .....	162
7.5 MCMC representativeness, accuracy, and efficiency .....	178
7.6 Summary .....	188
7.7 Exercises .....	189
<b>8. JAGS</b> .....	<b>193</b>
8.1 JAGS and its relation to R .....	193
8.2 A complete example .....	195
8.3 Simplified scripts for frequently used analyses .....	206
8.4 Example: difference of biases .....	208
8.5 Sampling from the prior distribution in JAGS .....	211
8.6 Probability distributions available in JAGS .....	213
8.7 Faster sampling with parallel processing in RunJAGS .....	215
8.8 Tips for expanding JAGS models .....	218
8.9 Exercises .....	218

<b>9. Hierarchical Models</b> .....	<b>221</b>
9.1 A single coin from a single mint .....	223
9.2 Multiple coins from a single mint .....	230
9.3 Shrinkage in hierarchical models .....	245
9.4 Speeding up JAGS .....	249
9.5 Extending the hierarchy: subjects within categories .....	251
9.6 Exercises .....	260
<b>10. Model Comparison and Hierarchical Modeling</b> .....	<b>265</b>
10.1 General formula and the Bayes factor .....	266
10.2 Example: two factories of coins .....	268
10.3 Solution by MCMC .....	274
10.4 Prediction: model averaging .....	289
10.5 Model complexity naturally accounted for .....	289
10.6 Extreme sensitivity to prior distribution .....	292
10.7 Exercises .....	295
<b>11. Null Hypothesis Significance Testing</b> .....	<b>297</b>
11.1 Paved with good intentions .....	300
11.2 Prior knowledge .....	315
11.3 Confidence interval and highest density interval .....	317
11.4 Multiple comparisons .....	325
11.5 What a sampling distribution is good for .....	329
11.6 Exercises .....	331
<b>12. Bayesian Approaches to Testing a Point ("Null") Hypothesis</b> .....	<b>335</b>
12.1 The estimation approach .....	336
12.2 The model-comparison approach .....	343
12.3 Relations of parameter estimation and model comparison .....	352
12.4 Estimation or model comparison? .....	354
12.5 Exercises .....	355
<b>13. Goals, Power, and Sample Size</b> .....	<b>359</b>
13.1 The will to power .....	360
13.2 Computing power and sample size .....	366
13.3 Sequential testing and the goal of precision .....	383
13.4 Discussion .....	393
13.5 Exercises .....	396

<b>14. Stan</b> .....	<b>399</b>
14.1 HMC sampling .....	400
14.2 Installing Stan .....	407
14.3 A complete example .....	407
14.4 Specify models top-down in Stan .....	414
14.5 Limitations and extras .....	415
14.6 Exercises .....	415
<b>PART III The Generalized Linear Model</b> .....	<b>417</b>
<b>15. Overview of the Generalized Linear Model</b> .....	<b>419</b>
15.1 Types of variables .....	420
15.2 Linear combination of predictors .....	423
15.3 Linking from combined predictors to noisy predicted data .....	435
15.4 Formal expression of the GLM .....	444
15.5 Exercises .....	446
<b>16. Metric-Predicted Variable on One or Two Groups</b> .....	<b>449</b>
16.1 Estimating the mean and standard deviation of a normal distribution .....	450
16.2 Outliers and robust estimation: the $t$ distribution .....	458
16.3 Two groups .....	468
16.4 Other noise distributions and transforming data .....	472
16.5 Exercises .....	473
<b>17. Metric Predicted Variable with One Metric Predictor</b> .....	<b>477</b>
17.1 Simple linear regression .....	478
17.2 Robust linear regression .....	479
17.3 Hierarchical regression on individuals within groups .....	490
17.4 Quadratic trend and weighted data .....	495
17.5 Procedure and perils for expanding a model .....	501
17.6 Exercises .....	504
<b>18. Metric Predicted Variable with Multiple Metric Predictors</b> .....	<b>509</b>
18.1 Multiple linear regression .....	510
18.2 Multiplicative interaction of metric predictors .....	525
18.3 Shrinkage of regression coefficients .....	530
18.4 Variable selection .....	536
18.5 Exercises .....	549

<b>19. Metric Predicted Variable with One Nominal Predictor</b> .....	<b>553</b>
19.1 Describing multiple groups of metric data .....	554
19.2 Traditional analysis of variance .....	556
19.3 Hierarchical Bayesian approach .....	557
19.4 Including a metric predictor .....	568
19.5 Heterogeneous variances and robustness against outliers .....	573
19.6 Exercises .....	579
<b>20. Metric Predicted Variable with Multiple Nominal Predictors</b> .....	<b>583</b>
20.1 Describing groups of metric data with multiple nominal predictors .....	584
20.2 Hierarchical Bayesian approach .....	588
20.3 Rescaling can change interactions, homogeneity, and normality .....	599
20.4 Heterogeneous variances and robustness against outliers .....	602
20.5 Within-subject designs .....	606
20.6 Model comparison approach .....	616
20.7 Exercises .....	618
<b>21. Dichotomous Predicted Variable</b> .....	<b>621</b>
21.1 Multiple metric predictors .....	622
21.2 Interpreting the regression coefficients .....	629
21.3 Robust logistic regression .....	635
21.4 Nominal predictors .....	636
21.5 Exercises .....	646
<b>22. Nominal Predicted Variable</b> .....	<b>649</b>
22.1 Softmax regression .....	650
22.2 Conditional logistic regression .....	655
22.3 Implementation in JAGS .....	659
22.4 Generalizations and variations of the models .....	667
22.5 Exercises .....	668
<b>23. Ordinal Predicted Variable</b> .....	<b>671</b>
23.1 Modeling ordinal data with an underlying metric variable .....	672
23.2 The case of a single group .....	675
23.3 The case of two groups .....	682
23.4 The case of metric predictors .....	685
23.5 Posterior prediction .....	698
23.6 Generalizations and extensions .....	699
23.7 Exercises .....	700

<b>24. Count Predicted Variable</b> .....	<b>703</b>
24.1 Poisson exponential model .....	704
24.2 Example: hair eye go again .....	711
24.3 Example: interaction contrasts, shrinkage, and omnibus test .....	713
24.4 Log-linear models for contingency tables .....	715
24.5 Exercises .....	715
<b>25. Tools in the Trunk</b> .....	<b>721</b>
25.1 Reporting a Bayesian analysis .....	721
25.2 Functions for computing highest density intervals .....	725
25.3 Reparameterization .....	729
25.4 Censored data in JAGS .....	732
25.5 What next? .....	736
<i>Bibliography</i> .....	737
<i>Index</i> .....	747