

**Michal Křížek Florian Luca
Lawrence Somer**

17 Lectures on Fermat Numbers

From Number Theory to Geometry

With a Foreword by Alena Šolcová

With 71 Illustrations



Springer

Michal Krížek
Mathematical Institute
Academy of Sciences
Prague 1, CZ-115 67
Czech Republic

Florian Luca
Mathematical Institute
National Autonomous University
of Mexico
Morelia, CP 58 089
Mexico

Lawrence Somer
Department of Mathematics
Catholic University of America
Washington, DC 20064
USA

Editors-in-Chief
Rédacteurs-en-chef

Jonathan Borwein
Peter Borwein
Centre for Experimental and Constructive Mathematics
Department of Mathematics and Statistics
Simon Fraser University
Burnaby, British Columbia V5A 1S6
Canada

Mathematics Subject Classification (2000): 11Axx, 11A41, 11Dxx, 11NO5, 11Rxx

Library of Congress Cataloging-in-Publication Data
Krizek, M.

17 lectures on Fermat numbers : from number theory to geometry / Michal Krizek,
Florian Luca, Lawrence Somer.

p. cm. — (CMS books in mathematics ; 9)

includes bibliographical references and index.

ISBN 978-1-4419-2952-5 ISBN 978-0-387-21850-2 (eBook)

DOI 10.1007/978-0-387-21850-2

I. Fermat numbers. I. Title: Seventeen lectures on Fermat numbers. II. Luca, Florian.
III. Somer, Lawrence. IV. Title. V. Series.

QA246.K75 2001

512'.7—dc21

2001042960

Printed on acid-free paper.

© 2001 Springer Science+Business Media New York
Originally published by Springer-Verlag New York Inc. in 2001
Softcover reprint of the hardcover 1st edition 2001

All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher Springer Science+Business Media, LLC, except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden.

The use of general descriptive names, trade names, trademarks, etc., in this publication, even if the former are not especially identified, is not to be taken as a sign that such names, as understood by the Trade Marks and Merchandise Marks Act, may accordingly be used freely by anyone.

Production managed by Terry Kornak; manufacturing supervised by Jerome Basma.
Photocomposed copy prepared from the authors' PostScript files.
Printed and bound by Edwards Brothers, Inc., Ann Arbor, MI.

9 8 7 6 5 4 3 2 1

SPIN 10844773

Contents

Foreword	vii
Preface	xix
Glossary of Symbols	xxi
1. Introduction	1
2. Fundamentals of Number Theory	9
3. Basic Properties of Fermat Numbers	27
4. The Most Beautiful Theorems on Fermat Numbers	34
5. Primality of Fermat Numbers	42
6. Divisibility of Fermat Numbers	60
7. Factors of Fermat Numbers	71
8. Connection of Fermat Numbers with Pascal's Triangle	81
9. Miscellaneous Results	96
10. The Irrationality of the Sum of Some Reciprocals	106
11. Fermat Primes and a Diophantine Equation	119
12. Fermat's Little Theorem, Pseudoprimes, and Superpseudoprimes	132
13. Generalizations of Fermat Numbers	149
14. Open Problems	160
15. Fermat Number Transform and Other Applications	167
16. The Proof of Gauss's Theorem	189
17. Euclidean Construction of the Regular Heptadecagon	195
Appendix	209
A. Tables of Fermat Numbers and Their Prime Factors	209
B. Mersenne Numbers	215
C. Remembrance of Pierre de Fermat	220
References	228

Web Site Sources 243
Name Index 244
Subject Index 249