

# **Applied Mathematical Sciences**

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*(continued following index)*

Rainer Kress

# Linear Integral Equations



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*To the Memory of My Parents*

# Preface

I fell in love with integral equations about twenty years ago when I was working on my thesis, and I am still attracted by their mathematical beauty. This book will try to stimulate the reader to share this love with me.

Having taught integral equations a number of times I felt a lack of a text which adequately combines theory, applications and numerical methods. Therefore, in this book I intend to cover each of these fields with the same weight. The first part provides the basic Riesz-Fredholm theory for equations of the second kind with compact operators in dual systems including all functional analytic concepts necessary for developing this theory. The second part then illustrates the classical applications of integral equation methods to boundary value problems for the Laplace and the heat equation as one of the main historical sources for the development of integral equations, and also introduces Cauchy type singular integral equations. The third part is devoted to describing the fundamental ideas for the numerical solution of integral equations. Finally, in a fourth part, ill-posed integral equations of the first kind and their regularization are studied in a Hilbert space setting.

In order to make the book accessible not only to mathematicians but also to physicists and engineers I have planned it as self-contained as possible by requiring only a solid foundation in differential and integral calculus and, for parts of the book, in complex function theory. Some background in functional analysis will be helpful, but the basic concepts of the theory of normed spaces will be briefly reviewed, and all functional analytic tools which are relevant in the study of integral equations will be developed in the book. Of course, I expect the reader to be willing to accept the functional analytic language for describing the theory and the numerical solution of integral equations. I hope that I succeeded in finding the adequate compromise between presenting integral equations in the proper modern framework and the danger of being too abstract.

An introduction to integral equations cannot present a complete picture of all classical aspects of the theory and of all recent developments. In this sense, this book intends to tell the reader what I think appropriate to teach students in a two-semester course on integral equations. I am willing to admit that the choice of a few of the topics might be biased by my own preferences and that some important subjects are omitted.

I am indebted to Dipl.-Math. Peter Hähner for carefully reading the book, for checking the solutions to the problems and for a number of suggestions for valuable improvements. Thanks also go to Frau Petra Trapp who spent some

time assisting me in the preparation of the L<sup>A</sup>T<sub>E</sub>X version of the text. And a particular note of thanks is given to my friend David Colton for reading over the book and helping me with the English language. Part of the book was written while I was on sabbatical leave at the Department of Mathematics at the University of Delaware. I gratefully acknowledge the hospitality.

Göttingen, April 1989

Rainer Kress

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