Steven G. Krantz Harold R. Parks

A Primer of Real Analytic Functions

Second Edition

Springer Science+Business Media, LLC

Steven G. Krantz Washington University Department of Mathematics St. Louis, MO 63130-4899 U.S.A. Harold R. Parks Oregon State University Department of Mathematics Corvallis, OR 97331-4605 U.S.A.

Library of Congress Cataloging-in-Publication Data

A CIP catalogue record for this book is available from the Library of Congress, Washington D.C., USA.

AMS Subject Classifications: Primary: 26E05, 30B10, 32C05; Secondary: 14P15, 26A99, 26B10, 26B40, 26E10, 30B40, 32C09, 35A10, 54C30

Printed on acid-free paper ©2002 Springer Science+Business Media New York Originally published by Birkhäuser Boston in 2002 Softcover reprint of the hardcover 2nd edition 2002





All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher (Springer Science+Business Media, LLC), except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden.

The use in this publication of trade names, trademarks, service marks and similar terms, even if they are not identified as such, is not to be taken as an expression of opinion as to whether or not they are subject to proprietary rights.

ISBN 978-1-4612-6412-5 ISBN 978-0-8176-8134-0 (eBook) SPIN 10846987 DOI 10.1007/978-0-8176-8134-0

Reformatted from the author's files by TEXniques, Inc., Cambridge, MA.

987654321

Contents

Preface to the Second Edition				
Pr	eface	to the First Edition	xi	
1	Elen	nentary Properties	1	
	1.1	Basic Properties of Power Series	1	
	1.2	Analytic Continuation	11	
	1.3	The Formula of Faà di Bruno	16	
	1.4	Composition of Real Analytic Functions	18	
	1.5	Inverse Functions	20	
2	Mul	tivariable Calculus of Real Analytic Functions	25	
	2.1	Power Series in Several Variables	25	
	2.2	Real Analytic Functions of Several Variables	29	
	2.3	The Implicit Function Theorem	35	
	2.4	A Special Case of the Cauchy–Kowalewsky Theorem	42	
	2.5	The Inverse Function Theorem	47	
	2.6	Topologies on the Space of Real Analytic Functions	50	
	2.7	Real Analytic Submanifolds	54	
		2.7.1 Bundles over a Real Analytic Submanifold	56	
	2.8	The General Cauchy–Kowalewsky Theorem	61	

3	Clas	sical Topics 6	7
	3.0	Introductory Remarks	7
	3.1	The Theorem of Pringsheim and Boas	8
	3.2	Besicovitch's Theorem	2
	3.3	Whitney's Extension and Approximation Theorems 7	5
	3.4	The Theorem of S. Bernstein	9
4	Som	e Questions of Hard Analysis 8	3
	4.1	Quasi-analytic and Gevrey Classes	3
	4.2	Puiseux Series	5
	4.3	Separate Real Analyticity	4
5	Res	Its Motivated by Partial Differential Equations 11	5
	5.1	Division of Distributions I	5
		5.1.1 Projection of Polynomially Defined Sets	7
	5.2	Division of Distributions II	6
	5.3	The FBI Transform	5
	5.4	The Paley–Wiener Theorem	4
6	Topi	rs in Geometry 15	1
	6.1	The Weierstrass Preparation Theorem	1
	6.2	Resolution of Singularities	6
	6.3	Lojasiewicz's Structure Theorem for Real Analytic Varieties 16	6
	6.4	The Embedding of Real Analytic Manifolds	1
	6.5	Semianalytic and Subanalytic Sets	7
		6.5.1 Basic Definitions	7
		6.5.2 Facts Concerning Semianalytic and Subanalytic Sets 17	9
		6.5.3 Examples and Discussion	1
		6.5.4 Rectilinearization	4
Bi	bliog	aphy 18	7
In	dex	20	3