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# Potential Theory Surveys and Problems

Proceedings of a Conference held in Prague, July 19–24, 1987

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TO THE MEMORY OF

PROFESSOR MARCEL B R E L O T (1903-1987)

PREFACE

Within the tradition of meetings devoted to potential theory, a conference on potential theory took place in Prague on 19 - 24 July 1987. There were 116 participants and 37 accompanying persons from 26 countries. Some participants were partially supported by IMU.

The Conference was organized by the Faculty of Mathematics and Physics, Charles University, with the collaboration of the Institute of Mathematics, Czechoslovak Academy of Sciences, the Department of mathematics, Czech University of Technology, the Union of Czechoslovak Mathematicians and Physicists, and the Czechoslovak Scientific and Technical Society. The Conference was held under the auspices of the Rector of Charles University, Professor Z. Češka, corresponding member of the Czechoslovak Academy of Sciences.

The Conference was concerned with various aspects of potential theory, including the applications of potential theory in other areas.

Thirteen one-hour survey lectures were delivered in the course of the Conference and 11 survey papers based on these lectures are included in the first part of the present volume. Totally 69 scientific communications from different branches of potential theory were presented during the Conference. The majority of them will appear in the Proceedings of the Conference to be published by Plenum Publishing Company.

On the occasion of the Conference, a collection of problems from potential theory was compiled and is included, with additional commentaries, in the second part of this volume.

Potential theory is nowadays a vast mathematical field, which in many respects truly reflects the state of the art in mathematics and mathematical analysis in particular. From the very beginning, potential theory whose roots are in physics, has developed into an independent domain with significant applications in the theory of functions, and in differential equations including their numerical treat-

ment, which has also disclosed unexpected connections with other branches of mathematics like probability theory, and which employs various methods of modern mathematics including topology and axiomatics.

The present volume may thus be of interest to mathematicians specializing in the above mentioned fields and also to everybody interested in the present state of potential theory as a whole.

Josef Král Ivan Netuka

Jaroslav Lukeš Jiří Veselý

Praha, October 1987

TABLE OF CONTENTS

**SURVEYS**

A. Ancona: Positive harmonic functions and hyperbolicity ..... 1

N. Boboc and Gh. Bucur: Order and convexity in potential theory 24

K.L. Chung: Probability methods in potential theory ..... 42

E. Fabes: Layer potential methods for boundary value problems on Lipschitz domains ..... 55

B. Fuglede: Fine potential theory ..... 81

W. Hansen: Balayage spaces - a natural setting for potential theory ..... 98

I. Laine: Axiomatic non-linear potential theories ..... 118

E.M. Landis: Application of the potential theory to the study of qualitative properties of solutions of the elliptic and parabolic equations ..... 133

M. Ohtsuka: Weighted extremal length and Beppo Levi functions .. 154

G.F. Roach: An introduction to iterative techniques for potential problems ..... 162

G. Wildenhain: Potential theory methods for higher order elliptic equations ..... 181

**COLLECTION OF PROBLEMS** 197

P 1: Problems on distortion under conformal mappings (J.L. Fernandez) ..... 198

P 2: On the Riesz representation of finely superharmonic functions (B. Fuglede) ..... 199

P 3: Nonlinear elliptic measures (J. Heinonen) ..... 202

P 4: Problems on a relation between measures and corresponding potentials (M. Kanda) ..... 205

P 5: Open problems connected with level sets of harmonic functions (B. Kawohl) ..... 207

P 6: On the extremal boundary of convex compact measures which represent a non-regular point in Choquet simplex (D.G. Keselman) ..... 211

P 7: The problem of construction of the harmonic space based on Choquet simplex (D.G. Keselman) ..... 214

P 8: The problem on quasi-interior in Choquet simplexes (D.G. Keselman) ..... 216

P 9: Boundary regularity and potential-theoretic operators (J. Král) ..... 220

P 10: Contractivity of the operator of the arithmetical mean (J. Král and D. Medková) ..... 223

P 11: Fine maxima (J. Král and I. Netuka) ..... 226

P 12: Repeated singular integrals (E.R. Love) ..... 229

P 13: Cofine potential theory (J. Lukeš and J. Malý) ..... 231

P 14: Essential and principal balayages (J. Lukeš and J. Malý) . 232

P 15: Local connectedness of the fine topology (J. Lukeš and J. Malý) ..... 234

P 16: On the Lusin-Menchoff property (J. Lukeš and J. Malý) .... 236

P 17: Relations between parabolic capacities (F.-Y. Maeda) ..... 238

P 18: Isovolumetric inequalities for the least harmonic majorant of $|x|^p$ (M. Sakai) ..... 240

The Copenhagen Problems (C 1 - C 15) ..... 242

Selected problems from collection "Research problems in complex analysis" (R 1 - R 14) ..... 254

Comments on Problems (added in October 1987) ..... 260

Scientific programme ..... 261

Participants of the Conference ..... 265