

PARALLEL ALGORITHMS FOR LINEAR MODELS

Advances in Computational Economics

VOLUME 15

SERIES EDITORS

Hans Amman, *University of Amsterdam, Amsterdam, The Netherlands*

Anna Nagurney, *University of Massachusetts at Amherst, USA*

EDITORIAL BOARD

Anantha K. Duraiappah, *European University Institute*

John Geweke, *University of Minnesota*

Manfred Gilli, *University of Geneva*

Kenneth L. Judd, *Stanford University*

David Kendrick, *University of Texas at Austin*

Daniel McFadden, *University of California at Berkeley*

Ellen McGrattan, *Duke University*

Reinhard Neck, *University of Klagenfurt*

Adrian R. Pagan, *Australian National University*

John Rust, *University of Wisconsin*

Berc Rustem, *University of London*

Hal R. Varian, *University of Michigan*

The titles published in this series are listed at the end of this volume.

Parallel Algorithms for Linear Models Numerical Methods and Estimation Problems

by

Erricos John Kontoghiorghes

Université de Neuchâtel, Switzerland



Springer Science+Business Media, LLC

Library of Congress Cataloging-in-Publication Data

Kontoghiorghes, Erricos John.

Parallel algorithms for linear models : numerical methods and estimation problems / by Erricos John Kontoghiorghes.

p. cm. -- (Advances in computational economics; v. 15)

Includes bibliographical references and indexes.

ISBN 978-1-4613-7064-2 ISBN 978-1-4615-4571-2 (eBook)

DOI 10.1007/978-1-4615-4571-2

1. Linear models (Statistics)--Data processing. 2. Parallel algorithms. I. Title. II. Series.

QA276 .K645 2000

519.5'35--dc21

99-056040

Copyright © 2000 by Springer Science+Business Media New York

Originally published by Kluwer Academic Publishers, New York in 1992

Softcover reprint of the hardcover 1st edition 1992

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system or transmitted in any form or by any means, mechanical, photo-copying, recording, or otherwise, without the prior written permission of the publisher, Springer Science + Business Media, LLC

Printed on acid-free paper.

To Laurence and Louisa

Contents

List of Figures	ix
List of Tables	xi
List of Algorithms	xiii
Preface	xv
1. LINEAR MODELS AND QR DECOMPOSITION	1
1 Introduction	1
2 Linear model specification	1
2.1 The ordinary linear model	2
2.2 The general linear model	7
3 Forming the QR decomposition	10
3.1 The Householder method	11
3.2 The Givens rotation method	13
3.3 The Gram–Schmidt orthogonalization method	16
4 Data parallel algorithms for computing the QR decomposition	17
4.1 Data parallelism and the MasPar SIMD system	17
4.2 The Householder method	19
4.3 The Gram–Schmidt method	21
4.4 The Givens rotation method	22
4.5 Computational results	23
5 QRD of large and skinny matrices	23
5.1 The CPP GAMMA SIMD system	24
5.2 The Householder QRD algorithm	25
5.3 QRD of skinny matrices	27
6 QRD of a set of matrices	29
6.1 Equal size matrices	29
6.2 Matrices with different number of columns	34
2. OLM NOT OF FULL RANK	39
1 Introduction	39
2 The QLD of the coefficient matrix	40
2.1 SIMD implementation	41
3 Triangularizing the lower trapezoid	43

3.1	The Householder method	43
3.2	The Givens method	46
4	Computing the orthogonal matrices	49
5	Discussion	54
3.	UPDATING AND DOWNDATING THE OLM	57
1	Introduction	57
2	Adding observations	58
2.1	The hybrid Householder algorithm	60
2.2	The Bitonic and Greedy Givens sequences	67
2.3	Updating with a block lower-triangular matrix	75
2.4	QRD of structured banded matrices	82
2.5	Recursive and linearly constrained least-squares	87
3	Adding exogenous variables	90
4	Deleting observations	92
4.1	Parallel strategies	94
5	Deleting exogenous variables	99
4.	THE GENERAL LINEAR MODEL	105
1	Introduction	105
2	Parallel algorithms	108
3	Implementation and performance analysis	111
5.	SURE MODELS	117
1	Introduction	117
2	The generalized linear least squares method	121
3	Triangular SURE models	123
3.1	Implementation aspects	127
4	Covariance restrictions	129
4.1	The QLD of the block bi-diagonal matrix	133
4.2	Parallel strategies	138
4.3	Common exogenous variables	140
6.	SIMULTANEOUS EQUATIONS MODELS	147
1	Generalized linear least squares	149
1.1	Estimating the disturbance covariance matrix	151
1.2	Redundancies	152
1.3	Inconsistencies	153
2	Modifying the SEM	154
3	Linear Equality Constraints	157
3.1	Basis of the null space and direct elimination methods	158
4	Computational Strategies	160
	References	163
	Author Index	177
	Subject Index	179

List of Figures

1.1	Geometric interpretation of least-squares for the OLM problem.	4
1.2	Illustration of Algorithm 1.2, where $m = 4$ and $n = 3$.	15
1.3	The column and diagonally based Givens sequences for computing the QRD.	15
1.4	Cyclic mapping of a matrix and a vector on the MasPar MP-1208.	18
1.5	Examples of Givens rotations schemes for computing the QRD.	22
1.6	Execution time ratio between 2-D and 3-D algorithms for computing the QRDs, where $G = 16$.	34
1.7	Stages of computing the QRDs (1.47).	36
2.1	Annihilation pattern of (2.4) using Householder reflections.	44
2.2	Givens sequences for computing the orthogonal factorization (2.4).	47
2.3	Illustration of the implementation phases of PGS, where $es = 4$.	49
2.4	The <i>fill-in</i> of the submatrix $P_{1:n,1:\bar{n}}$ at each phase of Algorithm 2.4.	53
3.1	Updating Givens sequences for computing the orthogonal factorizations (3.6), where $k = 8$ and $n = 4$.	59
3.2	Ratio of the execution times produced by the models of the <i>cyclic-layout</i> and <i>column-layout</i> implementations.	63
3.3	Computing (3.21) using Givens rotations.	71
3.4	The <i>bitonic</i> algorithm, where $n = 6$, $k = 18$ and $p_1 = p_2 = p_3 = 6$.	72
3.5	The <i>Greedy</i> sequence for computing (3.6a), where $n = 6$ and $k = 18$.	73

3.6	Computing the factorization (3.23) using the <i>diagonally</i> -based method, where $G = 5$.	76
3.7	Parallel strategies for computing the factorization (3.24)	77
3.8	Computing factorization (3.23).	78
3.9	The <i>column</i> -based method using the UGS-2 scheme.	80
3.10	The <i>column</i> -based method using the <i>Greedy</i> scheme.	81
3.11	Illustration of the annihilation patterns of <i>method-1</i> .	83
3.12	Computing (3.31) for $b = 8$, $\vartheta^* = 3$ and $j = 2$. Only the affected matrices are shown.	85
3.13	Illustration of <i>method-3</i> , where $p = 4$ and $g = 1$.	86
3.14	Givens parallel strategies for downdating the QRD.	96
3.15	Illustration of the SK-based scheme for computing the QRD of RS .	102
3.16	<i>Greedy</i> -based schemes for computing the QRD of RS .	104
4.1	Sequential Givens sequences for computing the QLD (4.3a).	107
4.2	The SK sequence.	109
4.3	$G^{(16)}B$ with $e(16, 18, 8) = 8$.	109
4.4	The application of the SK sequence to compute (4.3) on a 2-D SIMD computer.	109
4.5	Examples of the $MSK(p)$ sequence for computing the QLD.	110
5.1	The correlations $\rho_{i,j}$ in the SURE-CC model for $\vartheta_i = i$ and $\vartheta_i = 1/i$.	131
5.2	Factorization process for computing the QLD (5.35) using Algorithm 5.3.	136
5.3	Annihilation sequences of computing the factorization (5.40).	137
5.4	Givens sequences of computing the factorization (5.45).	138
5.5	Number of CDGRs for computing the orthogonal factorization (5.40) using the PDS.	139
5.6	Annihilation sequence of triangularizing (5.55).	144
6.1	Givens sequence for computing the QRD of RS_i .	161

List of Tables

1.1	Times (in seconds) of computing the QRD of a $128M \times 64N$ matrix.	23
1.2	Execution times (in seconds) of the CPP LALIB QR_FACTOR subroutine and the BPHA.	27
1.3	Execution times (in seconds) of the CPP LALIB QR_FACTOR subroutine and Algorithm 1.9.	28
1.4	Times (in seconds) of simultaneously computing the QRDs (1.47).	33
1.5	The <i>task-farming</i> and <i>scattering</i> methods for computing the QRDs (1.47).	38
2.1	Computing the QLD (2.3) (in seconds), where $m = Mes$ and $n = Nes$.	44
2.2	The CDGRs of the PGS for computing the factorization (2.4).	47
2.3	Computing (2.4) (in seconds), where $k = Kes$ and $n - k = \eta es$.	50
2.4	Times (in seconds) of reconstructing the orthogonal matrices Q^T and P on the DAP.	54
3.1	Execution times (msec) of the Householder and Givens methods for updating the QRD on the DAP.	60
3.2	Execution times (in seconds) for $k = 11264$.	63
3.3	Execution times (in seconds) of the RLS Householder algorithm on the MasPar.	65
3.4	Execution times (in seconds) of the RLS Householder algorithm on the GAMMA.	66
3.5	Number of CDGRs required to compute the factorization (3.6a).	74
3.6	Times (in seconds) for computing the orthogonal factorization (3.6a).	74

3.7	Computing the QRD of a structured banded matrix using <i>method-3</i> .	87
3.8	Estimated time (msec) required to compute $\hat{x}^{(i)}$ ($i = 2, 3, \dots$), where $m_i = 96$, $n = 32N$ and $k = 32K$.	91
3.9	Execution time (in seconds) for downdating the OLM.	98
4.1	Execution times (in seconds) of the MSK($\lambda \mathbf{es}/2$).	114
4.2	Computing (4.3) (in seconds) without explicitly constructing Q^T and P .	115
5.1	Computing (5.24), where $T - k - 1 = \tau \mathbf{es}$, $G - 1 = \mu \mathbf{es}$ and $\mathbf{es} = 32$.	128
5.2	Execution times of Algorithm 5.2 for solving $R\Gamma = \Delta$.	129

List of Algorithms

1.1	Computing the QRD of $A \in \mathfrak{R}^{m \times n}$ using Householder transformations.	12
1.2	The column-based Givens sequence for computing the QRD of $A \in \mathfrak{R}^{m \times n}$.	14
1.3	The diagonally-based Givens sequence for computing the QRD of $A \in \mathfrak{R}^{m \times n}$.	16
1.4	The Classical Gram-Schmidt method for computing the QRD of $A \in \mathfrak{R}^{m \times n}$.	16
1.5	The Modified Gram-Schmidt method for computing the QRD.	17
1.6	QR factorization by Householder transformations on SIMD systems.	20
1.7	The MGS method for computing the QRD on SIMD systems.	21
1.8	The CPP LALIB method for computing the QR Decomposition.	26
1.9	Householder with parallelism in the first dimension.	28
1.10	The Householder algorithm.	30
1.11	The Modified Gram-Schmidt algorithm.	31
1.12	The <i>task-farming</i> approach for computing the QRDs (1.47) on p ($p \ll G$) processors using a SPMD paradigm.	37
2.1	The QL decomposition of A .	43
2.2	Triangularizing the lower trapezoid using Householder reflections.	45
2.3	The reconstruction of the orthogonal matrix Q in (2.3).	51
2.4	The reconstruction of the orthogonal matrix P in (2.4).	53
3.1	The data-parallel Householder algorithm.	61
3.2	The <i>bitonic</i> algorithm for updating the QRD, where $\hat{R} \equiv R_{\lambda+1}^{(0)}$.	70
3.3	The computation of (3.63) using Householder transformations.	97
5.1	An iterative algorithm for solving tSURE models.	126

5.2	The parallel solution of the triangular system $R\Gamma = \Delta$.	129
5.3	Computing the QLD (5.35).	135

Preface

The monograph provides a complete and detailed account of the design, analysis and implementation of parallel algorithms for solving large-scale linear models. It investigates and presents efficient, numerically stable algorithms for computing the least-squares estimators and other quantities of interest on massively parallel systems.

The least-squares computations are based on orthogonal transformations, in particular the QR and QL decompositions. Parallel algorithms employing Givens rotations and Householder transformations have been designed for various linear model estimation problems. Some of the algorithms presented are parallel versions of serial methods while others are original designs. The implementation of the major parallel algorithms is described. The necessary techniques and insights needed for implementing efficient parallel algorithms on multiprocessor systems are illustrated in detail. Although most of the algorithms have been implemented on SIMD systems the data parallel computations of these algorithms should, in general, be applicable to any massively parallel computer.

The monograph is in two parts. The first part consists of four chapters and deals with the computational aspects for solving linear models that have applicability in diverse areas. The remaining two chapters form the second part which concentrates on numerical and computational methods for solving various problems associated with seemingly unrelated regression equations (SURE) and simultaneous equations models.

Chapter 1 provides a brief introduction to linear models and considers various forms for solving the QR decomposition on serial and parallel systems. Emphasis is given to the design and efficient implementation of the parallel algorithms. The second chapter investigates the performance and practical issues for solving the ordinary linear model (OLM), with the exogenous matrix being ill-conditioned or having deficient rank, on a SIMD system.

Chapter 3 is devoted to methods for up- and down-dating the OLM. It provides the necessary computational tools and techniques that are often required in econometrics and optimization. The efficient parallel strategies for modifying the OLM can be used as primitives for designing fast econometric algorithms. For example, the Givens and Householder algorithms used to compute the QR decomposition after rows have been added or columns have been deleted from the original matrix have been efficiently employed to the solution of the SURE and simultaneous equations models. The updating methods are also employed to solve the recursive ordinary linear model with linear equality constraints. The numerical methods based on the basis of the null space and direct elimination methods are in turn adopted for the solution of linearly constrained simultaneous equations models.

The fourth chapter investigates parallel algorithms for solving the general linear model – the parent model of econometrics – when it is considered as a generalized linear least-squares problem. This approach has subsequently been efficiently used to compute solutions of SURE and simultaneous equations models without having as prerequisite the non-singularity of the variance-covariance matrix of the disturbances. Chapter 5 presents a parallel algorithm for solving triangular SURE models. The problem of computing estimates of parameters in SURE models with variance inequalities and positivity of correlations constraints is also considered. Finally, chapter 6 presents algorithms for computing the three-stage least squares estimator of simultaneous equations models (SEMs). Numerical and computational methods for solving SEMs with *separable* linear equalities constraints and when the SEM has been modified by deleting or adding new observations or variables are discussed. Expressions revealing linear combinations between the observations which become redundant are also presented.

These novel computational methods for solving SURE and simultaneous equations models provide new insights that can be useful to econometric modelling. Furthermore, the computational and numerical efficient treatment of these models, which are regarded as the core of econometric theory, can be considered as the basis for future research. The algorithms can be extended or modified to deal with models that occur in particular econometric applications and have specific characteristics that need to be taken into account.

The practical issues of the parallel algorithms and the theoretical aspects of the numerical methods will be of interest to a broad range of researchers working in the areas of numerical and computational methods in statistics and econometrics, parallel numerical algorithms, parallel computing and numerical linear algebra. The aim of this monograph is to promote research in the interface of econometrics, computational statistics, numerical linear algebra and parallelism.

The research described in this monograph is based on the work that I have pursued in the last ten years. During this period I was privileged to have the opportunity to discuss various issues related to my work with Maurice Clint. His numerous suggestions and constructive comments have been both inspiring and invaluable. I am grateful to Dennis Parkinson for his valuable information that he has provided on many occasions on various aspects related to SIMD systems, David A. Belsley for his constructive comments and advice on the solution of SURE and simultaneous equations models, Hans-Heinrich Nägeli for his comments and constructive criticism on performance issues of parallel algorithms and the late Mike R.B. Clarke for his suggestions on Givens sequences and matrix computations. I am indebted to Paolo Foschi and Manfred Gilli for their comments on this monograph and to Sharon Silverne for proof reading the manuscript. The author accepts full responsibility for any errors that may be found in this work.

Some of the results of this monograph were originally published in various papers [69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 84, 85, 86, 87, 88] and reproduced by kind permission of Elsevier Science Publishers B.V. © 1993, 1994, 1995, 1999; Gordon and Breach Publishers © 1993, 1995; John Wiley & Sons Limited © 1996, 1999; IEEE © 1993; Kluwer Academic Publishers © 1997, 1999; Principia Scientia © 1996, 1997; SAP-Slovak Academic Press Ltd. © 1995; and Springer-Verlag © 1993, 1996, 1999.