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Parallel Algorithms for Linear Models

Numerical Methods and Estimation Problems

by

Erricos John Kontoghiorghes
Université de Neuchâtel, Switzerland

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To Laurence and Louisa

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Preface

The monograph provides a complete and detailed account of the design, analysis and implementation of parallel algorithms for solving large–scale linear models. It investigates and presents efficient, numerically stable algorithms for computing the least–squares estimators and other quantities of interest on massively parallel systems.

The least–squares computations are based on orthogonal transformations, in particular the QR and QL decompositions. Parallel algorithms employing Givens rotations and Householder transformations have been designed for various linear model estimation problems. Some of the algorithms presented are parallel versions of serial methods while others are original designs. The implementation of the major parallel algorithms is described. The necessary techniques and insights needed for implementing efficient parallel algorithms on multiprocessor systems are illustrated in detail. Although most of the algorithms have been implemented on SIMD systems the data parallel computations of these algorithms should, in general, be applicable to any massively parallel computer.

The monograph is in two parts. The first part consists of four chapters and deals with the computational aspects for solving linear models that have applicability in diverse areas. The remaining two chapters form the second part which concentrates on numerical and computational methods for solving various problems associated with seemingly unrelated regression equations (SURE) and simultaneous equations models.

Chapter 1 provides a brief introduction to linear models and considers various forms for solving the QR decomposition on serial and parallel systems. Emphasis is given to the design and efficient implementation of the parallel algorithms. The second chapter investigates the performance and practical issues for solving the ordinary linear model (OLM), with the exogenous matrix being ill–conditioned or having deficient rank, on a SIMD system.

Chapter 3 is devoted to methods for up– and down–dating the OLM. It provides the necessary computational tools and techniques that are often required in econometrics and optimization. The efficient parallel strategies for modifying the OLM can be used as primitives for designing fast econometric algorithms. For example, the Givens and Householder algorithms used to compute the QR decomposition after rows have been added or columns have been deleted from the original matrix have been efficiently employed to the solution of the SURE and simultaneous equations models. The updating methods are also employed to solve the recursive ordinary linear model with linear equality constraints. The numerical methods based on the basis of the null space and direct elimination methods are in turn adopted for the solution of linearly constrained simultaneous equations models.

The fourth chapter investigates parallel algorithms for solving the general linear model – the parent model of econometrics – when it is considered as a generalized linear least–squares problem. This approach has subsequently been efficiently used to compute solutions of SURE and simultaneous equations models without having as prerequisite the non–singularity of the variance–covariance matrix of the disturbances. Chapter 5 presents a parallel algorithm for solving triangular SURE models. The problem of computing estimates of parameters in SURE models with variance inequalities and positivity of correlations constraints is also considered. Finally, chapter 6 presents algorithms for computing the three–stage least squares estimator of simultaneous equations models (SEMs). Numerical and computational methods for solving SEMs with *separable* linear equalities constraints and when the SEM has been modified by deleting or adding new observations or variables are discussed. Expressions revealing linear combinations between the observations which become redundant are also presented.

These novel computational methods for solving SURE and simultaneous equations models provide new insights that can be useful to econometric modelling. Furthermore, the computational and numerical efficient treatment of these models, which are regarded as the core of econometric theory, can be considered as the basis for future research. The algorithms can be extended or modified to deal with models that occur in particular econometric applications and have specific characteristics that need to be taken into account.

The practical issues of the parallel algorithms and the theoretical aspects of the numerical methods will be of interest to a broad range of researchers working in the areas of numerical and computational methods in statistics and econometrics, parallel numerical algorithms, parallel computing and numerical linear algebra. The aim of this monograph is to promote research in the interface of econometrics, computational statistics, numerical linear algebra and parallelism.

The research described in this monograph is based on the work that I have pursued in the last ten years. During this period I was privileged to have the opportunity to discuss various issues related to my work with Maurice Clint. His numerous suggestions and constructive comments have been both inspiring and invaluable. I am grateful to Dennis Parkinson for his valuable information that he has provided on many occasions on various aspects related to SIMD systems, David A. Belsley for his constructive comments and advice on the solution of SURE and simultaneous equations models, Hans-Heinrich Nägeli for his comments and constructive criticism on performance issues of parallel algorithms and the late Mike R.B. Clarke for his suggestions on Givens sequences and matrix computations. I am indebted to Paolo Foschi and Manfred Gilli for their comments on this monograph and to Sharon Silverne for proof reading the manuscript. The author accepts full responsibility for any errors that may be found in this work.

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