Mathematics of the 19th Century

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Translated from the Russian
by Roger Cooke

Geometry

Analytic Function Theory

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Preface

The general principles by which the editors and authors of the present edition have been guided were explained in the preface to the first volume of *Mathematics of the 19th Century*, which contains chapters on the history of mathematical logic, algebra, number theory, and probability theory (Nauka, Moscow 1978; English translation by Birkhäuser Verlag, Basel-Boston-Berlin 1992). Circumstances beyond the control of the editors necessitated certain changes in the sequence of historical exposition of individual disciplines. The second volume contains two chapters: history of geometry and history of analytic function theory (including elliptic and Abelian functions); the size of the two chapters naturally entailed dividing them into sections. The history of differential and integral calculus, as well as computational mathematics, which we had planned to include in the second volume, will form part of the third volume.

We remind our readers that the appendix of each volume contains a list of the most important literature and an index of names. The names of journals are given in abbreviated form and the volume and year of publication are indicated; if the actual year of publication differs from the nominal year, the latter is given in parentheses. The book *History of Mathematics from Ancient Times to the Early Nineteenth Century* [in Russian], which was published in the years 1970–1972, is cited in abbreviated form as HM (with volume and page number indicated). The first volume of the present series is cited as Bk. 1 (with page numbers).

The first chapter of this volume was read by Prof. P. K. Rashevskiĭ, and the second by Prof. E. D. Solomentsev; the entire volume was read by Prof. A. D. Solov'ev. The authors and the editors are grateful to all three for useful advice and corrections. We also wish to express our gratitude to Prof. Taton (France), Dr. E. Fellmann (Switzerland), and K. Jacobs (Federal Republic of Germany) who kindly provided us with a number of portraits.

The first volume of *Mathematics of the 19th Century* was published at the same time as the *Abrégé d'histoire des mathématiques, 1700–1900*, edited by J. Dieudonné (Hermann: Paris, 1978, T. 1, 2).

The editors announce with deep sorrow that after the manuscript of this book had been completed one of its authors passed away. Academician Alekseĭ Ivanovich Markushevich (2.4.1908–4.6.1979) of the Academy of Pedagogical Sciences of the

USSR was one of the editors of the first two volumes in the series. Articles and obituaries dedicated to the memory of this prominent scholar and activist in the field of popular education have appeared in many Soviet journals.

It should be remarked that Markushevich had communicated certain important conclusions of his research, relating in particular to the connections between the doctoral dissertation of B. Riemann and the papers of A. Cauchy and V. Puiset immediately preceding it, in a paper given at the International Congress of Mathematicians in Helsinki (1978) and published in expanded form in Issue XXV of *Историко-Математические Исследования* (1980). These conclusions were subsequently confirmed completely by the Zürich scholar E. Neuenschwander, who studied the papers of Riemann from the period of his doctoral dissertation, which are kept in the archives of the University of Göttingen.

Moscow, 3 June 1980

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