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240. Elliott: Probabilistic Number Theory II
241. Rudin: Function Theory in the Unit Ball of C^n
242. Huppert/Blackburn: Finite Groups I
243. Huppert/Blackburn: Finite Groups II
244. Kubert/Lang: Modular Units

continued after Index

Kunihiko Kodaira

Complex Manifolds and Deformation of Complex Structures

Translated by Kazuo Akao

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Kunihiko Kodaira
3-19-8 Nakaochiai
Shinjuku-Ku, Tokyo
Japan

Kazuo Akao (*Translator*)
Department of Mathematics
Gakushuin University
Tshima-ku, Tokyo
Japan

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Dedicated to my esteemed colleague and friend
D. C. Spencer

Preface

This book is an introduction to the theory of complex manifolds and their deformations.

Deformation of the complex structure of Riemann surfaces is an idea which goes back to Riemann who, in his famous memoir on Abelian functions published in 1857, calculated the number of effective parameters on which the deformation depends. Since the publication of Riemann's memoir, questions concerning the deformation of the complex structure of Riemann surfaces have never lost their interest.

The deformation of algebraic surfaces seems to have been considered first by Max Noether in 1888 (M. Noether: *Anzahl der Modulen einer Classe algebraischer Flächen*, Sitz. Königlich. Preuss. Akad. der Wiss. zu Berlin, erster Halbband, 1888, pp. 123–127). However, the deformation of higher dimensional complex manifolds had been curiously neglected for 100 years. In 1957, exactly 100 years after Riemann's memoir, Frölicher and Nijenhuis published a paper in which they studied deformation of higher dimensional complex manifolds by a differential geometric method and obtained an important result. (A. Frölicher and A. Nijenhuis: A theorem on stability of complex structures, *Proc. Nat. Acad. Sci., U.S.A.*, **43** (1957), 239–241).

Inspired by their result, D. C. Spencer and I conceived a theory of deformation of compact complex manifolds which is based on the primitive idea that, since a compact complex manifold M is composed of a finite number of coordinate neighbourhoods patched together, its deformation would be a shift in the patches. Quite naturally it follows from this idea that an infinitesimal deformation of M should be represented by an element of the cohomology group $H^1(M, \Theta)$ of M with coefficients in the sheaf Θ of germs of holomorphic vector fields. However, there seemed to be no reason that any given element of $H^1(M, \Theta)$ represents an infinitesimal deformation of M. In spite of this, examination of familiar examples of compact complex manifolds M revealed a mysterious phenomenon that $\dim H^1(M, \Theta)$ coincides with the number of effective parameters involved in the definition of M. In order to clarify this mystery, Spencer and I developed the theory of deformation of compact complex manifolds. The process of the development was the most interesting experience in my whole mathematical life. It was similar to an experimental science developed by the interaction between experiments (examination of examples) and theory. In this book I have tried to reproduce this interesting experience; however I could not fully convey it. Such an experience may be a passing phenomenon which cannot be reproduced.

The theory of deformation of compact complex manifolds is based on the theory of elliptic partial differential operators expounded in the Appendix. I would like to express my deep appreciation to Professor D. Fujiwara who kindly wrote the Appendix and also to Professor K. Akao who spent the time and effort translating this book into English.

Tokyo, Japan
January, 1985

KUNIHIKO KODAIRA

Contents