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Shoshichi Kobayashi   Transformation Groups in Differential Geometry

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Preface

Given a mathematical structure, one of the basic associated mathematical objects is its automorphism group. The object of this book is to give a biased account of automorphism groups of differential geometric structures. All geometric structures are not created equal; some are creations of gods while others are products of lesser human minds. Amongst the former, Riemannian and complex structures stand out for their beauty and wealth. A major portion of this book is therefore devoted to these two structures.

Chapter I describes a general theory of automorphisms of geometric structures with emphasis on the question of when the automorphism group can be given a Lie group structure. Basic theorems in this regard are presented in §§ 3, 4 and 5. The concept of G-structure or that of pseudo-group structure enables us to treat most of the interesting geometric structures in a unified manner. In § 8, we sketch the relationship between the two concepts. Chapter I is so arranged that the reader who is primarily interested in Riemannian, complex, conformal and projective structures can skip §§ 5, 6, 7 and 8. This chapter is partly based on lectures I gave in Tokyo and Berkeley in 1965.

Contents of Chapters II and III should be fairly clear from the section headings. It should be pointed out that the results in §§ 3 and 4 of Chapter II will not be used elsewhere in this book and those of §§ 5 and 6 of Chapter II will be needed only in §§ 10 and 12 of Chapter III. I lectured on Chapter II in Berkeley in 1968; Chapter II is a faithful version of the actual lectures.

Chapter IV is concerned with automorphisms of affine, projective and conformal connections. We treat both the projective and the conformal cases in a unified manner.

Throughout the book, we use *Foundations of Differential Geometry* as our standard reference. Some of the referential results which cannot be found there are assembled in Appendices for the convenience of the reader.

As its title indicates, this book is concerned with the differential geometric aspect rather than the differential topological or homological aspect of the theory of transformation groups. We have confined ourselves to presenting only basic results, avoiding difficult theorems. To compensate for the omission of many interesting but difficult results, we have supplied the reader with an extensive list of references.

We have not touched upon homogeneous spaces, partly because they form an independent discipline of their own. While we are interested in automorphisms of given geometric structures, the differential geometry of homogeneous spaces is primarily concerned with geometric objects which are invariant under given transitive transformation groups. For the convenience of the reader, the Bibliography includes papers on the geometry of homogeneous spaces which are related to the topics discussed here.

In concluding this preface, I would like to express my appreciation to a number of mathematicians: Professors Yano and Lichnerowicz, who interested me in this subject through their lectures, books and papers; Professor Ehresmann, who taught me jets, prolongations and infinite pseudo-groups; K. Nomizu, T. Nagano and T. Ochiai, my friends and collaborators in many papers; Professor Matsushima, whose recent monograph on holomorphic vector fields influenced greatly the presentation of Chapter III; Professor Howard, who kindly made his manuscript on holomorphic vector fields available to me. I would like to thank Professor Remmert and Dr. Peters for inviting me to write this book and for their patience.

I am grateful also to the National Science Foundation for its unfailing support given to me during the preparation of this book.

January, 1972 S. Kobayashi

Contents