



# Hyperbolic Manifolds and Holomorphic Mappings

An Introduction  
(Second Edition)

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**An Introduction**

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