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PREFACE

The core of this book is the author's thesis, Algebraic Spaces, written under Michael Artin at the Massachusetts Institute of Technology. The object there as here was to work out the foundations à la EGA for the theory of algebraic spaces, and hence give the necessary background for Artin's fundamental papers Algebraization of Formal Moduli I, II.

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Donald Knutson

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