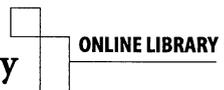


Elements of Newtonian Mechanics

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Jens M. Knudsen Poul G. Hjorth

Elements of Newtonian Mechanics

Including Nonlinear Dynamics

Third Revised and Enlarged Edition

With 341 Figures and 131 Problems with Solutions
and 89 Worked Examples



Springer

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Library of Congress Cataloging-in-Publication Data

Knudsen, J.M. (Jens M.), 1955-
Elements of Newtonian mechanics: including nonlinear dynamics / Jens M. Knudsen,
Poul G. Hjorth. - 3rd rev. and enl. ed.
p. cm. - (Advanced texts in physics, ISSN 1439-2674)
Includes bibliographical references and index.
ISBN 978-3-540-67652-2 ISBN 978-3-642-57234-0 (eBook)
DOI 10.1007/978-3-642-57234-0
1. Mechanics. 2. Mechanics--Problems, exercises, etc. I. Hjorth, P.G. (Poul G.), 1930-
II. Title. III. Series.
QC125.2 .K48 2000 531 -- dc21 00-044012

Third Edition 2000. Corrected 2nd Printing 2002

ISSN 1439-2674

ISBN 978-3-540-67652-2

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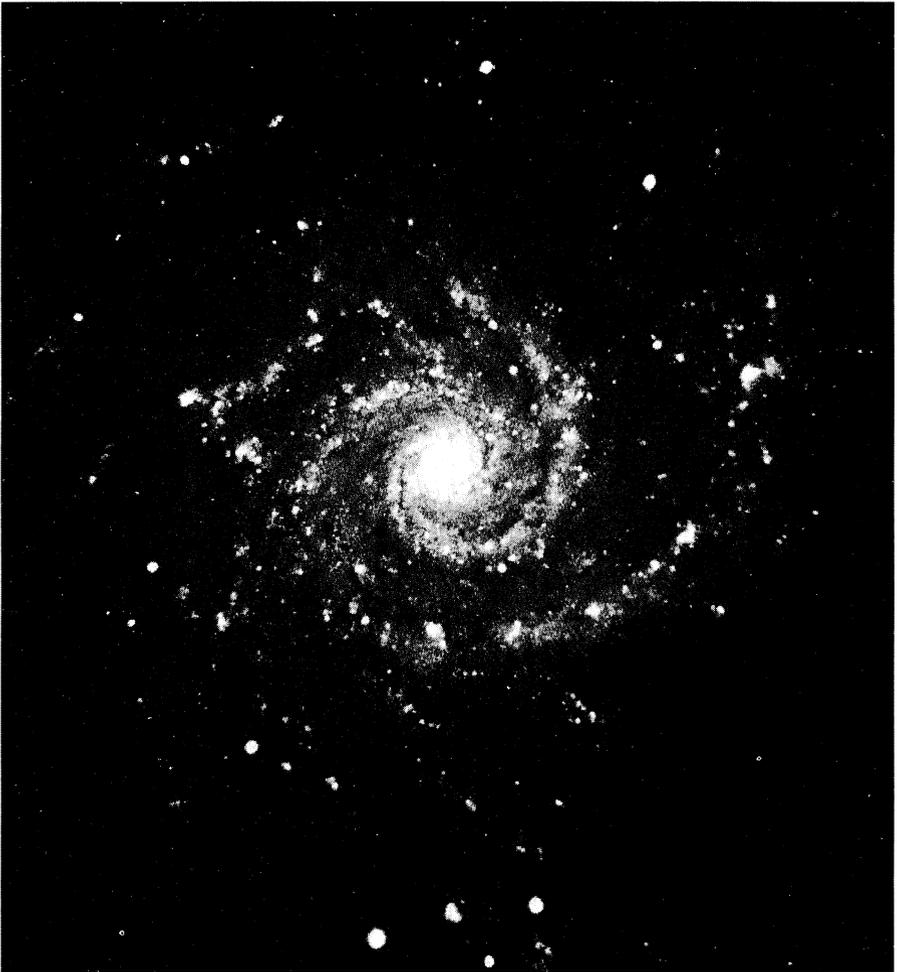
© Springer-Verlag Berlin Heidelberg 1995, 1996, 2000
Originally published by Springer-Verlag Berlin Heidelberg New York in 2000

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Typesetting: Data prepared by the author using a Springer \TeX macro package
Cover design: *design & production* GmbH, Heidelberg

Printed on acid-free paper SPIN 10914670 56/3111 5 4 3 2 1

*There is no physical experiment
by which you can determine
and later recover
a single point in astronomical space*



A Galaxy (M-74) in the constellation *Pisces*. This galaxy is about 20 000 000 light years from Earth. This is how our own galaxy, the Milky Way, would appear if seen from a great distance (Photograph © Hale Observatories, Pasadena)

A Note for the Reader

Nearly half of this book consists of examples. These examples form an absolutely essential part of the text and should by no means be skipped. The authors believe that Newton's ideas on the subtle problem of motion can be assimilated only by working with Newton's equations applied to concrete examples.

Listening to lectures is not enough. All processes of learning are somehow connected to active participation, and the learning of physics is no exception. To underline this viewpoint we have, at the beginning of the course, always written on the blackboard, as a kind of motto:

*At Home
by Your Desk.*

Nearly all the chapters in the book are followed by a set of problems. Very few of these problems are simple "plug-in" exercises. Most problems will demand some independent thinking. If you cannot solve all the problems at first try, do not despair. We have good advice which has worked for many students: study the text, and in particular the examples, one, two, ... many times over. In the end, you will succeed.

For several problems you will need some parameters, e.g., the mass of the Earth, the distance of the Earth from the Sun, etc. The numbers you need can all be found on the inside of the cover of the book. Answers to the problems are found at the back of the book.

Throughout the book SI units have been used. Relevant SI units can be found on the inside of the cover.

A Note for the Instructor

In Copenhagen, less than one quarter of the examples were presented in the lectures. Most of the remaining examples were assigned for recitation sessions.

JMK & PGH

Preface to the Third Edition

In the third edition a number of minor misprints that appeared in the second edition have been corrected. Furthermore, 17 new problems have been added, at the end of chapters 6, 8, 9, 11, 12, 13, and 14. The answers to these 17 problems have not been listed in the 'Answers' section at the end of the book. This will permit the problems to be used as hand-in problems or perhaps in mid-term exams.

Copenhagen
May 2000

JMK & PGH

Preface to the Second Edition

In the second edition, a number of misprints that appeared in the first edition have been corrected. In addition to this, we have made improvements based on the experience gathered in the use of the first English edition of the book in the introductory course in physics at the University of Copenhagen.

A chapter introducing nonlinear dynamics has been added. The purpose of this chapter is to provide supplementary reading for the students who are interested in this area of active research, where Newtonian mechanics plays an essential role. The students who wish to dig deeper, should consult texts dedicated to the study of nonlinear dynamical systems and chaos. The literature list at the end of this book contains several references for the topic.

The book still contains a one-semester (15 weeks) first university course on Newtonian mechanics. This necessarily introduces some constraints on the choice of topics and the level of mathematical sophistication expected from the reader. If one looks for discussions of technical issues, such as the physics behind various manifestations of friction, or the tensorial nature of the rotation vector, one will look in vain. The book contains what we feel are the essential aspects of Newtonian Mechanics.

It is a pleasure again to thank Springer-Verlag and in particular Dr. H. J. Kölsch and the staff at the Heidelberg office for helpfulness and professional collaboration.

Copenhagen
June 1996

JMK & PGH

Preface to the First Edition

This book is intended as a textbook for an entry-level university course in Newtonian mechanics for students of physics, astronomy, and the engineering sciences. The material has been used as a first-semester text for first-year undergraduates at the Niels Bohr Institute, which is part of the University of Copenhagen.

Our way of presenting Newtonian mechanics is influenced by the writings of the late Max Born. Also, the *Feynman Lectures on Physics* have been an important source of inspiration. In fact, the idea for the book came when we read Section 16.1 of Volume 1 of the *Feynman Lectures*. Ideas from the well-known *Berkeley Physics Course* may also be traced in the text. All of the books quoted in the literature list have, in one way or another, served as a source for our lectures for undergraduates.

It is assumed that the students already have a rudimentary knowledge of Newtonian mechanics, say at the high-school level. Some background in *vectors* and *elementary calculus* is also required, i.e., the students should know how to add vectors as well as how to differentiate and integrate elementary functions. The Appendix contains the required background for the use of vectors in Newtonian mechanics.

Careful study of the many worked examples will give the student the ability to use the powerful tools of Newtonian mechanics. Furthermore, we emphasize the fundamental problem of motion from the very beginning. This prepares the students for an understanding of Einstein's special and general theory of relativity. The text will demonstrate for the student that the answer to the question "What does it mean that a body moves?" is far from simple.

The authors wish to express their gratitude to many colleagues for discussions and for the encouragement given. Special thanks goes to J. Lyngesen who has read the entire manuscript and provided many valuable suggestions.

It is a pleasure for the authors to thank cand. polyt. E. B. Beran at the Technical University of Denmark, B. Kaluza and Blue Sky Research (makers of Textures) for assistance in L^AT_EX typesetting, and Springer-Verlag Heidelberg, in particular Dr. H.J. Kölsch, for excellent cooperation in the preparation of the manuscript.

Copenhagen
January 1995

*Jens Martin Knudsen
Poul Georg Hjorth*

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