

Kunio Murasugi

Translated by Bohdan Kurpita

KNOT THEORY
and
ITS APPLICATIONS

Kunio Murasugi
Department of Mathematics
University of Toronto
Toronto, Ontario M5S 1A1
Canada

Bohdan Kurpita
The Daiwa Anglo-Japanese
Foundation and Waseda University
Shinjuku-ku, Tokyo
Japan

Library of Congress Cataloging-in-Publication Data

Murasugi, Kunio, 1929-

[Musubime riron to sono ōyō. English]

Knot theory and its applications / Kunio Murasugi ; translated by
Bohdan Kurpita.

p. cm.

Includes bibliographical references (p. -) and index.

ISBN 978-0-8176-4718-6

paper)

1. Knot theory. I. Title.

QA612.2.M8613 1996

514'.224--dc20

96-16329

CIP

Printed on acid-free paper

Published originally in 1993 in Japanese

Birkhäuser



© Birkhäuser Boston 1996

Softcover reprint of the hardcover 1st edition 1996

Copyright is not claimed for works of U.S. Government employees.

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, without prior permission of the copyright owner.

Permission to photocopy for internal or personal use of specific clients is granted by Springer Science+Business Media, LLC for libraries and other users registered with the Copyright Clearance Center (CCC), provided that the base fee of \$6.00 per copy, plus \$0.20 per page is paid directly to CCC, 222 Rosewood Drive, Danvers, MA 01923, U.S.A. Special requests should be addressed directly to Springer Science+Business Media, LLC

ISBN 978-0-8176-4718-6

ISBN 978-0-8176-4719-3 (eBook)

DOI 10.1007/978-0-8176-4719-3

Typeset in T_EX by Bohdan Kurpita

9 8 7 6 5 4 3 2

Contents

	Introduction	1
	Fundamental Concepts of Knot Theory	5
	1 The elementary knot moves	6
	2 The equivalence of knots (I)	7
	3 The equivalence of knots (II)	9
	4 Links	14
	5 Knot decomposition and the semi-group of a knot	17
	6 The cobordism group of knots	23
	Knot Tables	25
	1 Regular diagrams and alternating knots	26
	2 Knot tables	30
	3 Knot graphs	34
	Fundamental Problems of Knot Theory	40
	1 Global problems	41
	2 Local problems	43
	Classical Knot Invariants	47
	1 The Reidemeister moves	48
	2 The minimum number of crossing points	56
	3 The bridge number	58
	4 The unknotting number	61
	5 The linking number	64
	6 The colouring number of a knot	69
	Seifert Matrices	75
	1 The Seifert surface	76
	2 The genus of a knot	80
	3 The Seifert matrix	83
	4 S-equivalence of Seifert matrices	89



Invariants from the Seifert matrix	104
1 <i>The Alexander polynomial</i>	105
2 <i>The Alexander - Conway polynomial</i>	108
3 <i>Basic properties of the Alexander polynomial</i>	116
4 <i>The signature of a knot</i>	122



Torus Knots	132
1 <i>Torus knots</i>	133
2 <i>The classification of torus knots (I)</i>	137
3 <i>The Seifert matrix of a torus knot</i>	141
4 <i>The classification of torus knots (II)</i>	143
5 <i>Invariants of torus knots</i>	148



Creating Manifolds from Knots	152
1 <i>Dehn surgery</i>	154
2 <i>Covering spaces</i>	159
3 <i>The cyclic covering space of a knot</i>	163
4 <i>A theorem of Alexander</i>	166



Tangles and 2-Bridge Knots	171
1 <i>Tangles</i>	172
2 <i>Trivial tangles (rational tangles)</i>	176
3 <i>2-bridge knots (rational knots)</i>	182
4 <i>Oriented 2-bridge knots</i>	194



The Theory of Braids	197
1 <i>Braids</i>	198
2 <i>The braid group</i>	201
3 <i>Knots and braids</i>	209
4 <i>The braid index</i>	214



The Jones Revolution	217
1 <i>The Jones polynomial</i>	219
2 <i>The basic characteristics of the Jones polynomial</i>	222
3 <i>The skein invariants</i>	231
4 <i>The Kauffman polynomial</i>	232
5 <i>The skein polynomials and classical knot invariants. (Alternating knots and the Tait conjectures)</i>	241



Knots via Statistical Mechanics	248
1 The 6-vertex model	249
2 The partition function for braids	255
3 An invariant of knots	260



Knot Theory in Molecular Biology	267
1 DNA and knots	268
2 Site-specific recombination	271
3 A model for site-specific recombination	273
4 Recombination due to the recombinase Tn3 Resolvase	276



Graph Theory Applied to Chemistry	284
1 An invariant of graphs: the chromatic polynomial	286
2 Bing's conjecture and spatial graphs	289
3 The chirality of spatial graphs	296



Vassiliev Invariants	300
1 Singular knots	301
2 Vassiliev invariants	304
3 Some examples of Vassiliev invariants	308
4 Chord diagrams	313
5 Final remarks	321

Appendix	325
Appendix (I): a table of knots	326
Appendix (II): Alexander and Jones polynomials	327

Notes	329
--------------------	------------

Bibliography	333
---------------------------	------------

Index	337
--------------------	------------