Mathematical Logic

Stephen Cole Kleene

DOVER PUBLICATIONS, INC. Mineola, New York

Copyright

Copyright © 1967 by Stephen Cole Kleene All rights reserved

Bibliographical Note

This Dover edition, first published in 2002, is an unabridged republication of the work originally published in 1967 by John Wiley & Sons, Inc., New York.

Library of Congress Cataloging-in-Publication Data

Kleene, Stephen Cole, 1909-

Mathematical logic / Stephen Cole Kleene.

p. cm,

Originally published New York Wiley, 1967 Includes bibliographical references and index.

ISBN 0-486-42533-9 (pbk)

1. Mathematics—Philosophy. 2. Logic, Symbolic and mathematical I. Title.

QA9 A1 K54 2002 511 3---dc21

2002034823

Manufactured in the United States of America Dover Publications, Inc., 31 East 2nd Street, Mineola, N.Y 11501

CONTENTS

PART I. ELEMENTARY MATHEMATICAL LOGIC

CHAPTER	R I. THE PROPOSITIONAL CALCULUS	3
§ 1.	Linguistic considerations; formulas	3
§ 2.	Model theory: truth tables, validity	8
§ 3.	Model theory; the substitution rule, a collection of valid	
	formulas	13
§ 4.	Model theory: implication and equivalence	17
§ 5.	Model theory: chains of equivalences	20
*§ 6.	Model theory: duality	22
§ 7.	Model theory: valid consequence	25
*§ 8.	Model theory: condensed truth tables	28
§ 9.	Proof theory: provability and deducibility	33
§ 10.	Proof theory: the deduction theorem	39
§ 11.	Proof theory: consistency, introduction and elimination	
	rules	43
§ 12.	Proof theory: completeness	45
§ 13.	Proof theory: use of derived rules	50
*§ 14.	Applications to ordinary language: analysis of arguments	58
*§ 15.	Applications to ordinary language: incompletely stated	
	arguments	67
СНАРТЕ	R II. THE PREDICATE CALCULUS	74
§ 16.	Linguistic considerations: formulas, free and bound	
•	occurrences of variables	74
§ 17.	Model theory: domains, validity	83
§ 18.	Model theory: basic results on validity	93
*§ 19.	Model theory: further results on validity	96
§ 20.	Model theory: valid consequence	101
§ 21.	Proof theory: provability and deducibility	107
§ 22.	Proof theory: the deduction theorem	112
§ 23.	Proof theory: consistency, introduction and elimination	
	rules	116
§ 24.	Proof theory: replacement, chains of equivalences	121
		хi

xii CONTENTS

§ 25.	Proof theory: alterations of quantifiers, prenex form	125
*§ 26.	Applications to ordinary language: sets, Aristotelian	
	categorical forms	134
*§ 27.	Applications to ordinary language: more on translating	
	words into symbols	140
CHAPTE	R III. THE PREDICATE CALCULUS WITH EQUALITY	148
*§ 28.	Functions, terms	148
	Equality	151
-	Equality vs. equivalence, extensionality	157
	Descriptions	167
PART		
	AND THE FOUNDATIONS OF MATHEMATICS	
CHAPTE	R IV. THE FOUNDATIONS OF MATHEMATICS	175
§ 32.	Countable sets	175
§ 33.	Cantor's diagonal method	180
§ 34.	Abstract sets	183
§ 35.	The paradoxes	186
§ 36.	Axiomatic thinking vs. intuitive thinking in mathematics	191
§ 37.	Formal systems, metamathematics	198
§ 38.	Formal number theory	201
*§ 39.	Some other formal systems	215
СНАРТЕ	r V. Computability and decidability	223
§ 40.	Decision and computation procedures	223
§ 41.	Turing machines, Church's thesis	232
§ 42.	Church's theorem (via Turing machines)	242
§ 43.	Applications to formal number theory: undecidability	
	(Church) and incompleteness (Gödel's theorem)	247
§ 44.	Applications to formal number theory: consistency proofs	
	(Gödel's second theorem)	254
*§ 45.	Application to the predicate calculus (Church, Turing)	260
*§ 46.	Degrees of unsolvability (Post), hierarchies (Kleene,	
	Mostowski).	265
*§ 47.	Undecidability and incompleteness using only simple	
	consistency (Rosser)	273
CHAPTE	er VI. THE PREDICATE CALCULUS (ADDITIONAL TOPICS)	283
§ 48.	Gödel's completeness theorem: introduction	283
§ 49.	Gödel's completeness theorem: the basic discovery	295

	CONTENTS	xiii
§ 50.	Gödel's completeness theorem with a Gentzen-type formal	
	system, the Löwenheim-Skolem theorem	305
§ 51.	Gödel's completeness theorem (with a Hilbert-type formal	
	system)	312
§ 52.	Gödel's completeness theorem, and the Löwenheim-Skolem	
	theorem, in the predicate calculus with equality	315
§ 53.	Skolem's paradox and nonstandard models of arithmetic	321
§ 54.	Gentzen's theorem	331
*§ 55.	Permutability, Herbrand's theorem	338
	Craig's interpolation theorem	349
§ 57.	Beth's theorem on definability, Robinson's consistency	
	theorem	361
BIBLIOG	BIBLIOGRAPHY	
THEORE	THEOREM AND LEMMA NUMBERS: PAGES	
LIST OF	LIST OF POSTULATES	
SYMBOLS AND NOTATIONS		
INDEX		