
Contents

	<i>page</i>
<i>Preface</i>	<i>ix</i>
1 Continued fractions: real numbers	1
1.1 Historical background	1
1.2 Euler's theory of continued fractions	11
1.3 Rational approximations	17
1.4 Jean Bernoulli sequences	36
1.5 Markoff sequences	49
2 Continued fractions: algebra	71
2.1 Euler's algorithm	71
2.2 Lagrange's theorem	81
2.3 Pell's equation	84
2.4 Equivalent irrationals	92
2.5 Markoff's theory	98
3 Continued fractions: analysis	123
3.1 Convergence: elementary methods	123
3.2 Contribution of Brouncker and Wallis	131
3.3 Brouncker's method and the gamma function	146
4 Continued fractions: Euler	158
4.1 Partial sums	158
4.2 Euler's version of Brouncker's method	163
4.3 An extension of Wallis' formula	169
4.4 Wallis' formula for sinusoidal spirals	174
4.5 An extension of Brouncker's formula	177
4.6 On the formation of continued fractions	180
4.7 Euler's differential method	183
4.8 Laplace transform of hyperbolic secant	191
4.9 Stieltjes' continued fractions	194
4.10 Continued fraction of hyperbolic cotangent	199
4.11 Riccati's equation	206

5	Continued fractions: Euler's influence	228
5.1	Bauer–Muir–Perron theory	229
5.2	From Euler to Scott–Wall	232
5.3	The irrationality of π	238
5.4	The parabola theorem	240
6	<i>P</i>-fractions	247
6.1	Laurent series	247
6.2	Convergents	253
6.3	Quadratic irrationals	258
6.4	Hypergeometric series	272
6.5	Stieltjes' theory	285
7	Orthogonal polynomials	296
7.1	Euler's problem	296
7.2	Quadrature formulas	298
7.3	Sturm's method	303
7.4	Chebyshev's approach to orthogonal polynomials	310
7.5	Examples of orthogonal polynomials	315
8	Orthogonal polynomials on the unit circle	322
8.1	Orthogonal polynomials and continued fractions	322
8.2	The Gram–Schmidt algorithm	336
8.3	Szegő's alternative	346
8.4	Erdős measures	356
8.5	The continuum of Schur parameters	360
8.6	Rakhmanov measures	364
8.7	Convergence of Schur's algorithm on \mathbb{T}	368
8.8	Nevai's class	371
8.9	Inner functions and singular measures	380
8.10	Schur functions of smooth measures	388
8.11	Periodic measures	390
	<i>Appendix Continued fractions, observations L. Euler (1739)</i>	426
	<i>References</i>	466
	<i>Index</i>	475