P-adic Deterministic and Random Dynamics

Contents

De	Dedication		
Foreword			
Acknowledgments			
1.	ON	APPLICATIONS OF <i>P</i> -ADIC ANALYSIS	1
2.	P-2	5	
	1	Ultrametric spaces	5
	2	Non-archimedean fields	8
	3	The field of <i>p</i> -adic numbers	9
	4	Tree-like structure of the <i>p</i> -adic numbers	13
	5	Extensions of the field of <i>p</i> -adic numbers	14
	6	Analysis in complete non-Archimedean fields	20
	7	Analytic functions	22
	8	Hensel's lemma	23
	9	Roots of unity	25
	10	Some facts from number theory	27
3.	P-	ADIC DYNAMICAL SYSTEMS	31
	1	Periodic points and their character	31
	2	Monomial dynamical systems	34
4.	PERTURBATION OF MONOMIAL SYSTEMS		71
	1	Existence of Fixed Points of a Perturbated System	71
	2	Cycles of Perturbed Systems	76

5.	DY	NAMICAL SYSTEMS IN FINITE EXTENSIONS OF \mathbb{Q}_P	83
	1	Some examples on behaviour of polynomial dynamical systems in finite extensions.	83
	2	Polynomial dynamical systems over local fields	91
6.	CO	NJUGATE MAPS	99
	1	Introduction	99
	2	Attracting fixed points	100
	3	Repelling fixed points	103
	4	Small denominators	104
	5	Neutrally stable fixed points in \mathbb{C}_p	114
7.	P- I	ADIC ERGODICITY	117
	1	Minimality.	117
	2	Unique ergodicity.	119
8.	P-	ADIC NEURAL NETWORKS	123
	1	Hierarchical synaptic potentials	126
	2	Multidimensional case	132
	3	Minimization algorithm of learning	136
	4	Parametric dynamical networks	141
	5	p-adic model for memory retrieval	147
9.	DY	NAMICS IN ULTRA-PSEUDOMETRIC SPACES	155
	1	Extension of the <i>p</i> -adic mental model: associations and ideas	155
	2	Dynamics in pseudometric spaces of sets	160
	3	Existence of attractors	164
	4	Thinking with constant sharpness of associations	166
	5	Thinking with increasing sharpness of associations	168
	6	Strong triangle inequality for Hausdorff's pseudometric	171
10	RA	NDOM DYNAMICS	173
	1	Introduction to the theory of random dynamical systems	174
	2	Random dynamics for monomial maps	175
	3	Definition of Markovian dynamics	179
	4	Conditions for Markovian dynamics	182
	5	Concluding remarks	190

viii

Contents		ix
11. DYNAMICS OF PROBABILITY DISTRIBUTIONS ON THE <i>P</i> -ADIC MENTAL SPACE		
1	Dynamics of body \rightarrow mind field	195
2	Dynamics of probability distributions	200
3	Diffusion Model for Dynamics of Mental State	203
4	Mental State as the Distribution of a <i>p</i> -adic Random Walk	205
12. ULTRAMETRIC WAVELETS AND THEIR APPLICATIONS		
1	Construction of the ultrametric space	211
2	The wavelet basis in $L^2(\mu, X)$	215
3	Pseudodifferential operators	217
4	Relation to wavelets on real line	221
5	Ultrametric wavelets as elementary mental fields	225
13. THEORY OF P-ADIC VALUED PROBABILITY		
1	Probability as limit of frequencies in the p-adic topology	231
2	p-adic valued ensemble probability	236
3	Measures	245
4	<i>p</i> -adic probability space	252
References		
Index		269