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V. P. Khavin N. K. Nikol'skiĭ (Eds.)

# Commutative Harmonic Analysis IV

Harmonic Analysis in  $\mathbb{R}^n$



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## Preface

In this volume of the series “Commutative Harmonic Analysis”, three points mentioned in the preface to the first volume are realized: 1) Multiple Fourier series and Fourier integrals; 2) The machinery of singular integrals; 3) Exceptional sets in harmonic analysis.

The first theme is the subject matter of the contribution by Sh. A. Ali-mov, R. R. Ashurov, A. K. Pulatov, which in an obvious way constitutes the “multidimensional parallel” to S. V. Kislyakov’s article in Volume I, devoted to the “inner” questions of Fourier analysis of functions of one variable. The passage to the analysis of functions defined on  $\mathbb{R}^n$ ,  $n > 1$ , tells us something essential about the nature of the problem under study.

The contribution by E. M. Dyn’kin, the beginning of which was already published in Volume I of this subseries, is devoted to singular integrals. Besides classical material (Calderón-Zygmund and Littlewood-Paley theory), this article contains an exposition of recent results, which in an essential way have widened the scope of the whole area and have made it possible to solve many old problems, thereby sometimes transcending the very frames of harmonic analysis in its canonical interpretation.

Quite different but highly interesting and often tantalizing material is collected in S. V. Kislyakov’s contribution, which concludes this volume, centering around the notion of “exceptional” (or “narrow”) sets (this topic was briefly discussed already in V. P. Khavin’s contribution in the first volume of this series). Special attention is given to the so-called Sidon set, which are connected with some important results obtained in recent years, and further to methods utilizing the notion of capacity, which has its origin in potential theory.

V. P. Khavin, N. K. Nikol’skiĭ

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