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Commutative Harmonic Analysis I

General Survey
Classical Aspects

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Preface

This volume is the first in the series devoted to the commutative harmonic analysis, a fundamental part of the contemporary mathematics. The fundamental nature of this subject, however, has been determined so long ago, that unlike in other volumes of this publication, we have to start with simple notions which have been in constant use in mathematics and physics. Planning the series as a whole, we have assumed that harmonic analysis is based on a small number of axioms, simply and clearly formulated in terms of group theory which illustrate its sources of ideas. However, our subject cannot be completely reduced to those axioms. This part of mathematics is so well developed and has so many different sides to it that no abstract scheme is able to cover its immense concreteness completely. In particular, it relates to an enormous stock of facts accumulated by the classical "trigonometric" harmonic analysis. Moreover, subjected to a general mathematical tendency of integration and diffusion of conventional intersubject borders, harmonic analysis, in its modern form, more and more rests on non-translation invariant constructions. For example, one of the most significant achievements of latter decades, which has substantially changed the whole shape of harmonic analysis, is the penetration in this subject of subtle techniques of singular integral operators. On the other hand, the traditional topics, such as studies of convolution equations, spectral theory of functions and ideals of convolution algebras, methods of theory of analytic functions in harmonic analysis on semigroups, etc., also occupy an important place in other surveys of this series as well as in harmonic analysis itself.

Below we list some of the topics which, we hope, will be covered in this series.

1. Methods and structure of commutative harmonic analysis. This article is included in the first volume and devoted to the foundations of harmonic analysis, a brief outline of its history, structure and connections with other subjects.
2. Classical themes in Fourier analysis. This is, in some sense, a guide to "trigonometric" Fourier analysis, where new achievements are given together with the results already included in a famous book of Zygmund $[89]_K$.*
3. Methods of singular integrals. Harmonic analysis in $\mathbb{R}^n$.
4. Multiple Fourier series and integrals.
5. Group-theoretic methods of commutative harmonic analysis.
6. Convolution equations and analysis of classical groups of translations.

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* $[\cdot]_K$ denotes a corresponding reference in S.V. Kislyakov's article.

7. Analysis of classical semigroups.
8. Tauberian theorems in harmonic analysis.
9. The uncertainty principle in harmonic analysis.
10. Probabilistic methods in harmonic analysis.
11. Exceptional sets.
12. Harmonic analysis in physics.

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