

# Heteroclinic cycles in thermal convection models

Nguyen Huu Khanh

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	Rayleigh-Bénard convection . . . . .	4
1.2	The Lorenz model and singular hyperbolic attractors . . . . .	7
1.3	Global bifurcations . . . . .	10
1.3.1	Homoclinic loops . . . . .	11
1.4	Codimension-two global bifurcations . . . . .	12
1.4.1	Neutral homoclinic loops . . . . .	12
1.4.2	Inclination flips . . . . .	13
1.5	Continuation of homoclinic and heteroclinic orbits . . . . .	13
1.6	Outline of the thesis . . . . .	15
<b>2</b>	<b>Derivation of the Busse model</b>	<b>19</b>
2.1	Differential equations for the modes . . . . .	21
2.2	The four modes Busse system . . . . .	25
<b>3</b>	<b>Numerical exploration</b>	<b>29</b>
3.1	Bifurcations of equilibria . . . . .	30
3.1.1	Pitchfork bifurcations . . . . .	31
3.1.2	Hopf bifurcations . . . . .	32
3.2	Bifurcation diagram . . . . .	32
3.3	Numerical implementation . . . . .	37
3.4	Heteroclinic bifurcations with double principal eigenvalues . . . . .	38
3.5	Neutral heteroclinic cycles . . . . .	41
3.6	Inclination flips with symmetry . . . . .	46
3.7	T-point heteroclinic cycles . . . . .	50
3.8	Generalized Hopf Bifurcation . . . . .	52
3.9	Bogdanov-Takens bifurcation with symmetry . . . . .	54
3.10	Heteroclinic cycles with pitchfork equilibria . . . . .	55
<b>4</b>	<b>Normal forms and exponential expansions</b>	<b>57</b>
4.1	Invariant manifolds and foliations . . . . .	57
4.2	Local normal forms . . . . .	58
4.2.1	Normal forms for equilibria with double principal stable eigenvalues . . . . .	59

4.2.2	Normal forms for equilibria with one single principal stable eigenvalue . . . . .	61
4.3	Exponential expansions . . . . .	62
4.3.1	Exponential expansions for double principal stable eigenvalues . . . . .	62
4.3.2	Exponential expansions for one single principal stable eigenvalue . . . . .	66
5	<b>Heteroclinic cycles with a double principal eigenvalue</b> . . . . .	69
5.1	Resonant heteroclinic cycles . . . . .	69
5.2	Return maps . . . . .	72
5.2.1	Singular rescalings . . . . .	75
5.3	Bifurcation equations . . . . .	77
5.4	Bifurcation curves . . . . .	78
5.4.1	Saddle-node bifurcation . . . . .	79
5.4.2	Symmetry breaking bifurcation . . . . .	79
5.4.3	Homoclinic and heteroclinic connections . . . . .	80
5.4.4	Heteroclinic connections to periodic orbits . . . . .	80
5.4.5	Lorenz type attractors . . . . .	82
6	<b>Neutral heteroclinic cycles</b> . . . . .	83
6.1	Neutral heteroclinic cycles . . . . .	84
6.2	Return maps . . . . .	86
6.2.1	Stable foliations . . . . .	87
6.2.2	Singular rescalings . . . . .	88
6.3	Bifurcation equations . . . . .	89
6.4	Bifurcation curves . . . . .	90
6.4.1	Heteroclinic connections . . . . .	90
6.4.2	Saddle-node bifurcations of periodic orbits . . . . .	90
6.4.3	Symmetry breaking bifurcations of periodic orbits . . . . .	91
6.5	Strange attractors . . . . .	91
6.5.1	Singular hyperbolic sets for $0 < a < 1$ . . . . .	91
6.5.2	Contractive Lorenz flows . . . . .	92
6.6	Neutral homoclinic loops . . . . .	92
7	<b>Other organizing centers</b> . . . . .	95
7.1	Inclination flips with symmetry . . . . .	95
7.1.1	Return maps . . . . .	98
7.1.2	Singular rescalings . . . . .	99
7.1.3	Bifurcation equations . . . . .	100
7.1.4	Bifurcation curves . . . . .	100
7.1.5	Dynamics of the interval map . . . . .	103
7.2	T-point heteroclinic cycles . . . . .	106
7.2.1	Transition maps . . . . .	107
7.2.2	Heteroclinic cycles containing the origin and $Q_1$ . . . . .	109
7.2.3	Homoclinic loop to $M_3$ . . . . .	109
7.2.4	Heteroclinic connections between $M_3$ and $\mathcal{R}_2(M_3)$ . . . . .	110
7.3	Heteroclinic cycles with pitchfork equilibria . . . . .	110

<b>8 Extensions of the Busse system</b>	<b>115</b>
8.1 The Busse system for larger aspect ratios . . . . .	115
8.2 The Busse system with a larger number of modes . . . . .	117
<b>A Coefficients in the equations for the Busse system</b>	<b>123</b>
<b>B Mathematica code for the Busse system</b>	<b>125</b>