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Alexander S. Kechris

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To Alexandra and Olympia

Preface

This book is based on some notes that I prepared for a class given at Caltech during the academic year 1991–92, attended by both undergraduate and graduate students. Although these notes underwent several revisions, which included the addition of a new chapter (Chapter V) and of many comments and references, the final form still retains the informal and somewhat compact style of the original version. So this book is best viewed as a set of lecture notes rather than as a detailed and scholarly monograph.

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Los Angeles
September 1994

Alexander S. Kechris

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Introduction

Descriptive set theory is the study of "**definable sets**" in **Polish** (i.e., separable completely metrizable) **spaces**. In this theory, sets are classified in hierarchies, according to the complexity of their definitions, and the structure of the sets in each level of these hierarchies is systematically analyzed.

In the beginning we have the **Borel sets**, which are those obtained from the open sets, of a given Polish space, by the operations of complementation and countable union. Their class is denoted by $\mathbf{B}$. This class can be further analyzed in a transfinite hierarchy of length ω_1 (= the first uncountable ordinal), the **Borel hierarchy**, consisting of the open, closed, F_σ (countable unions of closed), G_δ (countable intersections of open), $F_{\sigma\delta}$ (countable intersections of F_σ), $G_{\delta\sigma}$ (countable unions of G_δ), etc., sets. In modern logical notation, these classes are denoted by $\mathbf{\Sigma}^0_\xi$, $\mathbf{\Pi}^0_\xi$, for $1 \leq \xi < \omega_1$, where

$$\mathbf{\Sigma}^0_1 = \text{ open, } \mathbf{\Pi}^0_1 = \text{ closed;}$$

$$\mathbf{\Sigma}^0_\xi = \{\bigcup_{n\in\mathbb{N}} A_n : A_n \text{ is in } \mathbf{\Pi}^0_{\xi_n} \text{ for } \xi_n < \xi\};$$

$$\mathbf{\Pi}^0_\xi = \text{ the complements of } \mathbf{\Sigma}^0_\xi \text{ sets.}$$

(Therefore, $\mathbf{\Sigma}^0_2 = F_\sigma$, $\mathbf{\Pi}^0_2 = G_\delta$, $\mathbf{\Sigma}^0_3 = G_{\delta\sigma}, \mathbf{\Pi}^0_3 = F_{\sigma\delta}$, etc.) Thus $\mathbf{B}$ ramifies in the following hierarchy:

$$\begin{matrix} \mathbf{\Sigma}^0_1 & \mathbf{\Sigma}^0_2 & & \mathbf{\Sigma}^0_\xi & & \mathbf{\Sigma}^0_\eta & \\ & & \cdots & & \cdots & & \cdots, \\ \mathbf{\Pi}^0_1 & \mathbf{\Pi}^0_2 & & \mathbf{\Pi}^0_\xi & & \mathbf{\Pi}^0_\eta & \end{matrix}$$

where $\xi \leq \eta < \omega_1$, every class is contained in any class to the right of it, and

$$\mathbf{B} = \bigcup_{\xi<\omega_1} \mathbf{\Sigma}^0_\xi = \bigcup_{\xi<\omega_1} \mathbf{\Pi}^0_\xi.$$

Beyond the Borel sets one has next the **projective sets**, which are those obtained from the Borel sets by the operations of projection (or continuous image) and complementation. The class of projective sets, denoted by **P**, ramifies in an infinite hierarchy of length ω (= the first infinite ordinal), the **projective hierarchy**, consisting of the analytic (**A**) (continuous images of Borel), co-analytic (**CA**) (complements of analytic), **PCA** (continuous images of **CA**), **CPCA** (complements of **PCA**), etc., sets. Again, in logical notation, we let

$$\begin{aligned} \mathbf{\Sigma}^1_1 &= \text{ analytic, } \mathbf{\Pi}^1_1 = \text{ co-analytic;} \\ \mathbf{\Sigma}^1_{n+1} &= \text{ all continuous images of } \mathbf{\Pi}^1_n \text{ sets;} \\ \mathbf{\Pi}^1_{n+1} &= \text{ the complements of } \mathbf{\Sigma}^1_{n+1} \text{ sets;} \end{aligned}$$

so that in the following diagram every class is contained in any class to the right of it:

$$\begin{matrix} & \mathbf{\Sigma}^1_1 & \mathbf{\Sigma}^1_2 & & \mathbf{\Sigma}^1_n & \mathbf{\Sigma}^1_{n+1} & \\ \mathbf{B} & & & \cdots & & & \cdots \\ & \mathbf{\Pi}^1_1 & \mathbf{\Pi}^1_2 & & \mathbf{\Pi}^1_n & \mathbf{\Pi}^1_{n+1} & \end{matrix}$$

and

$$\mathbf{P} = \bigcup_n \mathbf{\Sigma}^1_n = \bigcup_n \mathbf{\Pi}^1_n.$$

One can of course go beyond the projective hierarchy to study transfinite extensions of it, and even more complex "definable sets" in Polish spaces, but we will restrict ourselves here to the structure theory of Borel and projective sets, which is the subject matter of classical descriptive set theory.

Descriptive set theory has been one of the main areas of research in set theory for almost a century now. Moreover, its concepts and results are being used in diverse fields of mathematics, such as mathematical logic, combinatorics, topology, real and harmonic analysis, functional analysis, measure and probability theory, potential theory, ergodic theory, operator algebras, and topological groups and their representations. The main aim of these lectures is to provide a basic introduction to classical descriptive set theory and give some idea of its connections or applications to other areas.

About This Book

These lectures are divided into five chapters. The first chapter sets up the context by providing an overview of the basic theory of Polish spaces. Many standard tools, such as the Baire category theory, are also introduced here. The second chapter deals with the theory of Borel sets. Among other things, methods of infinite games figure prominently here, a feature that continues in the later chapters. In the third chapter, the theory of analytic sets, which is briefly introduced in the second chapter, is developed in more detail. The fourth chapter is devoted to the theory of co-analytic sets and, in particular, develops the machinery associated with ranks and scales. Finally, in the fifth chapter, we provide an introduction to the theory of projective sets, including the periodicity theorems.

We view this book as providing a first basic course in classical descriptive set theory, and we have therefore confined it largely to "core material" with which mathematicians interested in the subject for its own sake or those that wish to use it in their own field should be familiar. Throughout the book, however, are pointers to the literature for topics not treated here. In addition, a brief summary at the book's end (Section 40) describes the main further directions of current research in descriptive set theory.

Descriptive set theory can be approached from many different viewpoints. Over the years, researchers in diverse areas of mathematics—logic and set theory, analysis, topology, probability theory, and others—have brought their own intuitions, concepts, terminology, and notation to the subject. We have attempted in these lectures to present a largely balanced approach, which combines many elements of each tradition.

We have also made an effort to present a wide variety of examples and applications in order to illustrate the general concepts and results of the theory. Moreover, over 400 exercises are included, of varying degrees of difficulty. Among them are important results as well as propositions and lemmas, whose proofs seem best to be left to the reader. A section at the end of these lectures contains hints to selected exercises.

This book is essentially self-contained. The only thing it requires is familiarity, at the beginning graduate or even advanced undergraduate level, with the basics of general topology, measure theory, and functional analysis, as well as the elements of set theory, including transfinite induction and ordinals. (See, for example, H. B. Enderton [1977], P. R. Halmos [1960a] or Y. N. Moschovakis [1994].) A short review of some standard set theoretic concepts and notation that we use is given in Appendices A and B. Appendix C explains some of the basic logical notation employed throughout the text. It is recommended that the reader become familiar with the contents of these appendices before reading the book and return to them as needed later on. On occasion, especially in some examples, applications, or exercises, we discuss material, drawn from various areas of mathematics, which does not fall under the preceding basic prerequisites. In such cases, it is hoped that a reader who has not studied these concepts before will at least attempt to get some idea of what is going on and perhaps look over a standard textbook in one of these areas to learn more about them. (If this becomes impossible, this material can be safely omitted.)

Finally, given the rather informal nature of these lectures, we have not attempted to provide detailed historical or bibliographical notes and references. The reader can consult the monographs by N. N. Lusin [1972], K. Kuratowski [1966], Y. N. Moschovakis [1980], as well as the collection by C. A. Rogers et al. [1980] in that respect. The *Ω-Bibliography of Mathematical Logic* (G. H. Müller, ed., Vol. 5, Springer-Verlag, Berlin, 1987) also contains an extensive bibliography.