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The Higher Infinite

Large Cardinals in Set Theory
from Their Beginnings

Second Edition

 Springer

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The first edition was published in 1994 by Springer-Verlag under the same title in the series
Perspectives in Mathematical Logic

First softcover printing 2009

ISBN 978-3-540-88866-6

e-ISBN 978-3-540-88867-3

DOI 10.1007/978-3-540-88867-3

Springer Monographs in Mathematics ISSN 1439-7382

Library of Congress Control Number: 2008940025

Mathematics Subject Classification (2000): 03E05, 03E15, 03E35, 03E55, 03E60

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Coverdesign: WMXDesign GmbH, Heidelberg

Printed on acid-free paper

9 8 7 6 5 4 3 2 1

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