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Olav Kallenberg

Foundations of Modern Probability

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Preface

Some thirty years ago it was still possible, as Loève so ably demonstrated, to write a single book in probability theory containing practically everything worth knowing in the subject. The subsequent development has been explosive, and today a corresponding comprehensive coverage would require a whole library. Researchers and graduate students alike seem compelled to a rather extreme degree of specialization. As a result, the subject is threatened by disintegration into dozens or hundreds of subfields.

At the same time the interaction between the areas is livelier than ever, and there is a steadily growing core of key results and techniques that every probabilist needs to know, if only to read the literature in his or her own field. Thus, it seems essential that we all have at least a general overview of the whole area, and we should do what we can to keep the subject together. The present volume is an earnest attempt in that direction.

My original aim was to write a book about "everything." Various space and time constraints forced me to accept more modest and realistic goals for the project. Thus, "foundations" had to be understood in the narrower sense of the early 1970s, and there was no room for some of the more recent developments. I especially regret the omission of topics such as large deviations, Gibbs and Palm measures, interacting particle systems, stochastic differential geometry, Malliavin calculus, SPDEs, measure-valued diffusions, and branching and superprocesses. Clearly plenty of fundamental and intriguing material remains for a possible second volume.

Even with my more limited, revised ambitions, I had to be extremely selective in the choice of material. More importantly, it was necessary to look for the most economical approach to every result I did decide to include. In the latter respect, I was surprised to see how much could actually be done to simplify and streamline proofs, often handed down through generations of textbook writers. My general preference has been for results conveying some new idea or relationship, whereas many propositions of a more technical nature have been omitted. In the same vein, I have avoided technical or computational proofs that give little insight into the proven results. This conforms with my conviction that the logical structure is what matters most in mathematics, even when applications is the ultimate goal.

Though the book is primarily intended as a general reference, it should also be useful for graduate and seminar courses on different levels, ranging from elementary to advanced. Thus, a first-year graduate course in measure-theoretic probability could be based on the first ten or so chapters, while the rest of the book will readily provide material for more advanced courses on various topics. Though the treatment is formally self-contained, as far as measure theory and probability are concerned, the text is intended for a rather sophisticated reader with at least some rudimentary knowledge of subjects like topology, functional analysis, and complex variables.

My exposition is based on experiences from the numerous graduate and seminar courses I have been privileged to teach in Sweden and in the United States, ever since I was a graduate student myself. Over the years I have developed a personal approach to almost every topic, and even experts might find something of interest. Thus, many proofs may be new, and every chapter contains results that are not available in the standard textbook literature. It is my sincere hope that the book will convey some of the excitement I still feel for the subject, which is without a doubt (even apart from its utter usefulness) one of the richest and most beautiful areas of modern mathematics.

Notes and Acknowledgments: My first thanks are due to my numerous Swedish teachers, and especially to Peter Jagers, whose 1971 seminar opened my eyes to modern probability. The idea of this book was raised a few years later when the analysts at Gothenburg asked me to give a short lecture course on "probability for mathematicians." Although I objected to the title, the lectures were promptly delivered, and I became convinced of the project's feasibility. For many years afterward I had a faithful and enthusiastic audience in numerous courses on stochastic calculus, SDEs, and Markov processes. I am grateful for that learning opportunity and for the feedback and encouragement I received from colleagues and graduate students.

Inevitably I have benefited immensely from the heritage of countless authors, many of whom are not even listed in the bibliography. I have further been fortunate to know many prominent probabilists of our time, who have often inspired me through their scholarship and personal example. Two people, Klaus Matthes and Gopi Kallianpur, stand out as particularly important influences in connection with my numerous visits to Berlin and Chapel Hill, respectively.

The great Kai Lai Chung, my mentor and friend from recent years, offered penetrating comments on all aspects of the work: linguistic, historical, and mathematical. My colleague Ming Liao, always a stimulating partner for discussions, was kind enough to check my material on potential theory. Early versions of the manuscript were tested on several groups of graduate students, and Kamesh Casukhela, Davorin Dujmovic, and Hussain Talibi in particular were helpful in spotting misprints. Ulrich Albrecht and Ed Slaminka offered generous help with software problems. I am further grateful to John Kimmel, Karina Mikhli, and the Springer production team for their patience with my last-minute revisions and their truly professional handling of the project.

My greatest thanks go to my family, who is my constant source of happiness and inspiration. Without their love, encouragement, and understanding, this work would not have been possible.

Olav Kallenberg
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Contents