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Statistical Physics of Fluids

Basic Concepts and Applications

With 52 Figures and 5 Tables

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To the memory of my mother

Preface

This book grew out of the senior level lecture course I teach at Delft University and which I have taught in recent years at Eindhoven University and the University of Utrecht. Numerous discussions with students and colleagues led me to the conclusion that in spite of the existence of excellent books on the statistical theory of fluids, there is a gap between the fundamental theory and application of its concepts and techniques to practical problems. This book is an attempt to at least partially fill it.

It is not intended to be a thorough and comprehensive review of liquid state theory, which would inevitably require invoking a large number of results without actual derivation. Rather I prefer to focus on the main physical ideas and mathematical methods of fluid theory, starting with the basic principles of statistical mechanics, and present a detailed derivation of results accompanied by an explanation of their physical meaning. The same approach applies to several specialized topics of the liquid state, most of which are recent developments and belong to the areas of my own activities and thus reflect my personal taste. Wherever possible, theoretical predictions are compared with available experimental and simulation data.

So, what you are holding in your hands is neither a textbook nor a monograph, but rather a combination of both. It can be classified as an advanced text for graduate students in physics and chemistry with research interests in the statistical physics of fluids, and as a monograph for a professional audience in various areas of soft condensed matter. It can also be used by industrial scientists for background information, and as an advanced text for self-study.

I gratefully acknowledge the assistance of my colleagues and friends at various stages of the work. Chap. 7 on Monte Carlo methods was written together with Iosif Dyadkin; his vision of the subject and extraordinary general physical intuition guided me for many years. Carlo Luijten placed at my disposal his computer programs for the density functional calculations of surface tension in one-component systems (Sect. 9.3) and binary mixtures (Sect. 13.4.1). I would like to express my gratitude to Jos Thijssen for his careful reading of the manuscript and for a number of very constructive criticisms.

In creating the book I benefited greatly from discussions with a number of colleagues. In particular, Rini van Dongen, Bob Evans, Vladimir Filinov, Daan Frenkel, Ken Hamilton, Gert-Jan van Heijst, Jouke Heringa, Geert Hofmans, Simon de Leeuw, Henk Lekkerkerker, Christopher Lowe, Carlo Luijten, Thijs Michels, Bela Mulder, Piet Schram, Berend Smit, Vladimir Vorobiev, and Ben Widom made many helpful comments and suggestions.

Delft, April 2001 *Vitaly Kalikmanov*

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