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# Problems in Mathematical Analysis II

Continuity and  
Differentiation

W. J. Kaczor  
M. T. Nowak



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# Contents

Preface	xi
Notation and Terminology	xiii
<b>Problems</b>	
Chapter 1. Limits and Continuity	
1.1. The Limit of a Function	3
1.2. Properties of Continuous Functions	9
1.3. Intermediate Value Property	14
1.4. Semicontinuous Functions	18
1.5. Uniform Continuity	24
1.6. Functional Equations	27
1.7. Continuous Functions in Metric Spaces	32
Chapter 2. Differentiation	
2.1. The Derivative of a Real Function	37
2.2. Mean Value Theorems	45
2.3. Taylor's Formula and L'Hospital's Rule	52
2.4. Convex Functions	61

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2.5. Applications of Derivatives	68
2.6. Strong Differentiability and Schwarz Differentiability	77
Chapter 3. Sequences and Series of Functions	
3.1. Sequences of Functions, Uniform Convergence	81
3.2. Series of Functions, Uniform Convergence	87
3.3. Power Series	96
3.4. Taylor Series	102
<b>Solutions</b>	
Chapter 1. Limits and Continuity	
1.1. The Limit of a Function	111
1.2. Properties of Continuous Functions	129
1.3. Intermediate Value Property	146
1.4. Semicontinuous Functions	160
1.5. Uniform Continuity	171
1.6. Functional Equations	181
1.7. Continuous Functions in Metric Spaces	198
Chapter 2. Differentiation	
2.1. The Derivative of a Real Function	211
2.2. Mean Value Theorems	233
2.3. Taylor's Formula and L'Hospital's Rule	245
2.4. Convex Functions	267
2.5. Applications of Derivatives	285
2.6. Strong Differentiability and Schwarz Differentiability	310
Chapter 3. Sequences and Series of Functions	
3.1. Sequences of Functions, Uniform Convergence	317
3.2. Series of Functions, Uniform Convergence	336
3.3. Power Series	355

<b>Contents</b>	ix
3.4. Taylor Series	372
Bibliography - Books	393
Index	397