

Dieter Jungnickel

Graphs, Networks and Algorithms

Third Edition

With 209 Figures and 9 Tables

 Springer

Dieter Jungnickel

Universität Augsburg
Institut für Mathematik
86135 Augsburg
Germany
E-mail: jungnickel@math.uni-augsburg.de

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Contents

*When we have not what we like,
we must like what we have.*

COMTE DE BUSSY-RABUTIN

Preface to the Third Edition	VII
Preface to the Second Edition	IX
Preface to the First Edition	XI
1 Basic Graph Theory	1
1.1 Graphs, subgraphs and factors	2
1.2 Paths, cycles, connectedness, trees	5
1.3 Euler tours	13
1.4 Hamiltonian cycles	15
1.5 Planar graphs	21
1.6 Digraphs	25
1.7 An application: Tournaments and leagues	28
2 Algorithms and Complexity	33
2.1 Algorithms	34
2.2 Representing graphs	36
2.3 The algorithm of Hierholzer	39
2.4 How to write down algorithms	41
2.5 The complexity of algorithms	43
2.6 Directed acyclic graphs	46
2.7 NP-complete problems	49
2.8 HC is NP-complete	53
3 Shortest Paths	59
3.1 Shortest paths	59
3.2 Finite metric spaces	61
3.3 Breadth first search and bipartite graphs	63
3.4 Shortest path trees	68
3.5 Bellman's equations and acyclic networks	70

3.6	An application: Scheduling projects	72
3.7	The algorithm of Dijkstra	76
3.8	An application: Train schedules	81
3.9	The algorithm of Floyd and Warshall	84
3.10	Cycles of negative length	89
3.11	Path algebras	90
4	Spanning Trees	97
4.1	Trees and forests	97
4.2	Incidence matrices	99
4.3	Minimal spanning trees	104
4.4	The algorithms of Prim, Kruskal and Boruvka	106
4.5	Maximal spanning trees	113
4.6	Steiner trees	115
4.7	Spanning trees with restrictions	118
4.8	Arborescences and directed Euler tours	121
5	The Greedy Algorithm	127
5.1	The greedy algorithm and matroids	127
5.2	Characterizations of matroids	129
5.3	Matroid duality	135
5.4	The greedy algorithm as an approximation method	137
5.5	Minimization in independence systems	144
5.6	Accessible set systems	148
6	Flows	153
6.1	The theorems of Ford and Fulkerson	153
6.2	The algorithm of Edmonds and Karp	159
6.3	Auxiliary networks and phases	169
6.4	Constructing blocking flows	176
6.5	Zero-one flows	185
6.6	The algorithm of Goldberg and Tarjan	189
7	Combinatorial Applications	209
7.1	Disjoint paths: Menger's theorem	209
7.2	Matchings: König's theorem	213
7.3	Partial transversals: The marriage theorem	218
7.4	Combinatorics of matrices	223
7.5	Dissections: Dilworth's theorem	227
7.6	Parallelisms: Baranyai's theorem	231
7.7	Supply and demand: The Gale-Ryser theorem	234

8	Connectivity and Depth First Search	239
8.1	k -connected graphs	239
8.2	Depth first search	242
8.3	2-connected graphs	245
8.4	Depth first search for digraphs	252
8.5	Strongly connected digraphs	253
8.6	Edge connectivity	258
9	Colorings	261
9.1	Vertex colorings	261
9.2	Comparability graphs and interval graphs	265
9.3	Edge colorings	268
9.4	Cayley graphs	271
9.5	The five color theorem	275
10	Circulations	279
10.1	Circulations and flows	279
10.2	Feasible circulations	282
10.3	Elementary circulations	289
10.4	The algorithm of Klein	295
10.5	The algorithm of Busacker and Gowen	299
10.6	Potentials and ε -optimality	302
10.7	Optimal circulations by successive approximation	311
10.8	A polynomial procedure REFINE	315
10.9	The minimum mean cycle cancelling algorithm	322
10.10	Some further problems	327
10.11	An application: Graphical codes	329
11	The Network Simplex Algorithm	343
11.1	The minimum cost flow problem	344
11.2	Tree solutions	346
11.3	Constructing an admissible tree structure	349
11.4	The algorithm	353
11.5	Efficient implementations	358
12	Synthesis of Networks	363
12.1	Symmetric networks	363
12.2	Synthesis of equivalent flow trees	366
12.3	Synthesizing minimal networks	373
12.4	Cut trees	379
12.5	Increasing the capacities	383

13	Matchings	387
13.1	The 1-factor theorem	387
13.2	Augmenting paths	390
13.3	Alternating trees and blossoms	394
13.4	The algorithm of Edmonds	400
13.5	Matching matroids	416
14	Weighted matchings	419
14.1	The bipartite case	420
14.2	The Hungarian algorithm	421
14.3	Matchings, linear programs, and polytopes	430
14.4	The general case	434
14.5	The Chinese postman	438
14.6	Matchings and shortest paths	442
14.7	Some further problems	449
14.8	An application: Decoding graphical codes	452
15	A Hard Problem: The TSP	457
15.1	Basic definitions	457
15.2	Lower bounds: Relaxations	460
15.3	Lower bounds: Subgradient optimization	466
15.4	Approximation algorithms	471
15.5	Upper bounds: Heuristics	477
15.6	Upper bounds: Local search	480
15.7	Exact neighborhoods and suboptimality	483
15.8	Optimal solutions: Branch and bound	489
15.9	Concluding remarks	497
A	Some NP-Complete Problems	501
B	Solutions	509
B.1	Solutions for Chapter 1	509
B.2	Solutions for Chapter 2	515
B.3	Solutions for Chapter 3	520
B.4	Solutions for Chapter 4	527
B.5	Solutions for Chapter 5	532
B.6	Solutions for Chapter 6	535
B.7	Solutions for Chapter 7	545
B.8	Solutions for Chapter 8	554
B.9	Solutions for Chapter 9	560
B.10	Solutions for Chapter 10	563
B.11	Solutions for Chapter 11	572
B.12	Solutions for Chapter 12	572
B.13	Solutions for Chapter 13	578
B.14	Solutions for Chapter 14	583

B.15 Solutions for Chapter 15	589
C List of Symbols	593
C.1 General Symbols	593
C.2 Special Symbols	595
References	601
Index	635