

# Contents

<b>Jürgen Jost: Nonlinear Dirichlet Forms</b>	<b>1</b>
1.1 Introduction . . . . .	1
1.2 Definition and properties of generalized Dirichlet forms . . . . .	3
1.3 Resolvents, semigroups, and variational aspects . . . . .	10
1.4 Convergence properties . . . . .	24
1.5 Generalized harmonic maps . . . . .	28
1.6 Appendix: Geometric notions . . . . .	41
<b>Wilfrid S. Kendall: From Stochastic Parallel Transport to Har- monic Maps</b>	<b>49</b>
2.1 Introduction . . . . .	49
2.1.1 Aim and objectives. . . . .	49
2.1.2 History and motivation . . . . .	49
2.1.3 Stochastic calculus formalism . . . . .	51
2.2 Stochastic differential geometry . . . . .	56
2.2.1 Intrinsic approach to semimartingales . . . . .	56
2.2.2 Lifting to the frame bundle . . . . .	56
2.2.3 The semimartingale zoo . . . . .	59
2.2.4 Intrinsic geometry . . . . .	68
2.2.5 Diffusions and statistical shape . . . . .	69
2.2.6 Geodesic variational formulae . . . . .	69
2.2.7 Distance from a fixed point . . . . .	70
2.2.8 Distance between two moving points (I) . . . . .	71
2.2.9 Distance between two moving points (II) . . . . .	71
2.3 Distance and semimartingales . . . . .	72
2.3.1 Distance of a $\Gamma$ -martingale from a point . . . . .	73

2.3.2	Probabilistic approaches to Picard theorems . . . . .	74
2.3.3	$\Gamma$ -martingale convergence theorem . . . . .	76
2.3.4	An “Ultimate” Liouville Theorem . . . . .	77
2.3.5	Picard little theorem for harmonic maps . . . . .	79
2.3.6	Limiting directions for Brownian motion . . . . .	79
2.3.7	Probabilistic proofs for Picard little theorems . . . . .	81
2.3.8	Life on the cut-locus . . . . .	83
2.3.9	Historical perspective . . . . .	83
2.3.10	Cut-locus correction by upper bound . . . . .	84
2.4	Coupling and parallel transport . . . . .	85
2.4.1	Distance between two $\Gamma$ -martingales. . . . .	87
2.4.2	Coupling Brownian motions in the case of nonnegative Ricci curvature . . . . .	89
2.4.3	Applications . . . . .	92
2.5	Harmonic map Dirichlet problem . . . . .	94
2.5.1	The rôle of convexity and coupling . . . . .	94
2.5.2	Construction of $\Gamma$ -martingales . . . . .	96
2.5.3	Existence using barycentres . . . . .	97
2.5.4	Other approaches . . . . .	99
2.5.5	Smoothness <i>via</i> gradient estimates . . . . .	100
2.5.6	Equivalences . . . . .	101
<b>Umberto Mosco: Dirichlet forms and self-similarity</b>		<b>117</b>
3.1	Introduction . . . . .	117
3.2	Self-similar fractals . . . . .	120
3.3	Variational fractals . . . . .	124
3.4	The Lagrangian metrics . . . . .	129
3.5	The intrinsic homogeneous structure . . . . .	134
3.6	Field operators and spectral asymptotics . . . . .	136
3.7	Local solutions . . . . .	139
3.8	Scaled Poincaré inequalities . . . . .	143
3.9	Nash inequalities and Morrey–Sobolev imbeddings . . . . .	147
<b>Michael Röckner: Stochastic analysis on configuration spaces: basic ideas and recent results</b>		<b>157</b>
4.1	Introduction . . . . .	157
4.2	Lifting the geometry from the base manifold to the configuration space . . . . .	159

4.2.1	Test functions, flows, directional derivatives . . . . .	161
4.2.2	Tangent bundle, gradient, vector fields and divergence . .	161
4.2.3	Characterization of the volume elements as the mixed Poisson measures . . . . .	163
4.2.4	Laplacian and classical pre-Dirichlet form . . . . .	168
4.3	Infinite dimensional analysis and Brownian motion on configura- tion spaces . . . . .	169
4.3.1	Dirichlet forms and operators . . . . .	169
4.3.2	Markov uniqueness, essential self-adjointness and heat semigroup . . . . .	171
4.3.3	Brownian motion . . . . .	179
4.3.4	Ergodicity in the free case . . . . .	183
4.4	Classical Dirichlet forms on configuration spaces w.r.t. general measures . . . . .	184
4.4.1	Closability, domains and Sobolev spaces . . . . .	185
4.4.2	Quasi-regularity and corresponding diffusions . . . . .	190
4.5	Intrinsic metric on configuration spaces . . . . .	193
4.5.1	Identification of the intrinsic metric for a class of measures	194
4.5.2	A Rademacher theorem . . . . .	194
4.5.3	Potential-theoretic consequences . . . . .	195
4.6	Gibbs measures on configuration spaces . . . . .	196
4.6.1	Grand canonical Gibbs measures . . . . .	197
4.6.2	Canonical Gibbs measures . . . . .	199
4.6.3	Closability of the corresponding Dirichlet forms . . . . .	201
4.7	Integration by parts characterization of canonical Gibbs measures	209
4.7.1	Ruelle measures . . . . .	210
4.7.2	Integration by parts characterization . . . . .	213
4.8	Infinite interacting particle systems . . . . .	216
4.8.1	Stochastic dynamics corresponding to Gibbs states . . .	216
4.8.2	Solutions of the martingale problem resp. of the corre- sponding "heuristic" SDE . . . . .	217
4.9	Ergodicity . . . . .	218
4.9.1	A general result on irreducibility resp. ergodicity . . . .	219
4.9.2	Applications to canonical Gibbs and Ruelle measures . .	222
<b>Karl-Theodor Sturm: The Geometric Aspect of Dirichlet Forms</b>		<b>233</b>
5.1	Introduction . . . . .	233
5.2	Estimates for Semigroups and Resolvents . . . . .	235

5.2.1	The Dirichlet Form . . . . .	236
5.2.2	Estimates for the Semigroup . . . . .	237
5.2.3	Estimates for the Resolvent . . . . .	240
5.3	Capacity Estimates and Applications . . . . .	240
5.3.1	Basic Properties of Quasi-regular Dirichlet Forms . . . . .	241
5.3.2	The Capacity Estimate . . . . .	245
5.3.3	Criteria for Polarity . . . . .	249
5.3.4	Criteria for Recurrence and Transience . . . . .	252
5.4	The Intrinsic Metric . . . . .	255
5.4.1	Basic Properties of the Intrinsic Metric . . . . .	255
5.4.2	Global Form Properties and Geometry at Infinity . . . . .	258
5.5	Examples . . . . .	260
5.5.1	The Laplacian on $\mathbb{R}^N$ . . . . .	260
5.5.2	Laplacians with Weights . . . . .	261
5.5.3	Elliptic and Subelliptic Operators on $\mathbb{R}^N$ . . . . .	267
5.5.4	Laplace-Beltrami Operators on Riemannian Manifolds . . . . .	268