

Palle E.T. Jorgensen

Analysis and Probability Wavelets, Signals, Fractals

*With graphics by Brian Treadway
58 figures and illustrations*

 Springer

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Drawing by the author, next page:

Wavelet algorithms are good for vast sets of numbers.

An engineering friend described the old approach to data mining as

“Just drop a computer down onto a gigantic set of unstructured numbers!”

(data mining: see Section 6.2, pp. 102–105, and the Glossary, pp. xxiv–xxv).