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For Mrs. H

Preface

"... many eminent scholars, endowed with great geometric talent, make a point of never disclosing the simple and direct ideas that guided them, subordinating their elegant results to abstract general theories which often have no application outside the particular case in question. Geometry was becoming a study of algebraic, differential or partial differential equations, thus losing all the charm that comes from its being an art."

H. Lebesgue, *Leçons sur les Constructions Géométriques*, Gauthier-Villars, Paris, 1949.

This book is based on lecture courses given to final-year students at the University of Nottingham and to M.Sc. students at the University of the West Indies in an attempt to reverse the process of expurgation of the geometry component from the mathematics curricula of universities. This erosion is in sharp contrast to the situation in research mathematics, where the ideas and methods of geometry enjoy ever-increasing influence and importance. In the other direction, more modern ideas have made a forceful and beneficial impact on the geometry of the ancients in many areas. Thus trigonometry has vastly clarified our concept of angle, calculus has revolutionised the study of plane curves, and group theory has become the language of symmetry.

To illustrate this last point at a fundamental level, consider the notion of congruence in plane geometry: two triangles are congruent if one can be moved onto the other so that they coincide exactly. This property is guaranteed by each of the familiar conditions SSS, SAS, SAA and RHS. So congruent triangles are just copies of the same triangle appearing in (possibly) different places. This makes it clear that congruence is an equivalence relation, whose three defining properties correspond to properties of the moves mentioned above:

- reflexivity – the identity move,
- symmetry – inverse moves,
- transitivity – composition of moves.

Thinking of these moves as transformations, for which the associative law holds automatically, we have precisely the four axioms for a group: closure, associativity, identity and inverses.

The group just described underlies and in a sense determines plane geometry. It is called the Euclidean group and occupies a dominant position in this book. Its elements are isometries, as defined in Chapter 1, and a detailed study of these occupies Chapters 2 and 4. The rather bulky Chapters 3 and 5 are intended as crash courses on the theory of groups and group presentations respectively, and both lay emphasis on groups that are semidirect products. Such groups arise in the classification of discrete subgroups of the Euclidean group in Chapters 6, 7 and 8, and corresponding tessellations (or tilings) appear in Chapter 9. Regular tessellations of the sphere are classified in Chapter 10, and tessellations of other spaces, such as the hyperbolic plane, form the subject of Chapter 11. Finally, the notions of polygon in 2-space and polyhedron in 3-space are generalised in Chapter 12 to that of a polytope in n-dimensional Euclidean space. Regular polytopes are then defined using group theory and classified in all dimensions. The classification contains some surprises in dimension 4 and is achieved by as elegant a piece of mathematics as you might imagine.

The exercises at the end of each chapter form an integral part of the book, being designed to reinforce your grasp of the material. A large majority are more or less routine, but a handful of more challenging problems are included for good measure. Solutions to most of them, or at least generous hints, are given later, and suggestions for background, alternative and further reading appear towards the end of the book.

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D.L.J.

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