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(North America):*

S. Axler  
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# Universitext

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*(continued after index)*

George A. Jennings

# Modern Geometry with Applications

With 150 figures



Springer

George A. Jennings  
Department of Mathematics  
California State University  
Dominguez Hills  
1000 E. Victoria Blvd.  
Carson, CA 90747 USA

*Editorial Board*

S. Axler  
Department of  
Mathematics  
Michigan State University  
East Lansing, MI 48824  
USA

F.W. Gehring  
Department of  
Mathematics  
University of Michigan  
Ann Arbor, MI 48109  
USA

P.R. Halmos  
Department of  
Mathematics  
Santa Clara University  
Santa Clara, CA 95053  
USA

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# Foreword

This book is an introduction to the theory and applications of “modern geometry” – roughly speaking, geometry that was developed after Euclid. It covers three major areas of non-Euclidean geometry and their applications: spherical geometry (used in navigation and astronomy), projective geometry (used in art), and spacetime geometry (used in the Special Theory of Relativity). In addition it treats some of the more useful topics from Euclidean geometry, focusing on the use of Euclidean motions, and includes a chapter on conics and the orbits of planets.

My aim in writing this book was to balance theory with applications. It seems to me that students of geometry, especially prospective mathematics teachers, need to be aware of how geometry is used as well as how it is derived. Every topic in the book is motivated by an application and many additional applications are given in the exercises. This emphasis on applications is responsible for a somewhat nontraditional choice of topics: I left out hyperbolic geometry, a traditional topic with practically no applications that are intelligible to undergraduates, and replaced it with the spacetime geometry of Special Relativity, a thoroughly non-Euclidean geometry with striking implications for our own physical universe. The book contains enough material for a one semester course in geometry at the sophomore-to-senior level, as well as many exercises, mostly of a non-routine nature (the instructor may want to supplement them with routine exercises of his/her own).

I prepared the illustrations on a PC using *Windows Draw 3.0* by Micrografx and *Mathematica 2.2* by Wolfram Research.

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