Undergraduate Texts in Mathematics

Springer Science+Business Media, LLC

Undergraduate Texts in Mathematics

Anglin: Mathematics: A Concise History and Philosophy.
Readings in Mathematics.
Anglin/Lambek: The Heritage of Thales.
Readings in Mathematics.
Apostol: Introduction to Analytic Number Theory. Second edition.
Armstrong: Basic Topology.
Armstrong: Groups and Symmetry.
Axler: Linear Algebra Done Right. Second edition.
Beardon: Limits: A New Approach to Real Analysis.
Bak/Newman: Complex Analysis. Second edition.
Banchoff/Wermer: Linear Algebra Through Geometry. Second edition.
Berberian: A First Course in Real Analysis.
Bix: Conics and Cubics: A Concrete Introduction to Algebraic Curves.
Brémaud: An Introduction to Probabilistic Modeling.
Bressoud: Factorization and Primality Testing.
Bressoud: Second Year Calculus.
Readings in Mathematics.
Brickman: Mathematical Introduction to Linear Programming and Game Theory.
Browder: Mathematical Analysis: An Introduction.
Buskes/van Rooij: Topological Spaces: From Distance to Neighborhood.
Callahan: The Geometry of Spacetime: An Introduction to Special and General Relavitity.
Carter/van Brunt: The Lebesgue–Stieltjes Integral: A Practical Introduction
Cederberg: A Course in Modern Geometries. Second edition.
Childs: A Concrete Introduction to Higher Algebra. Second edition.
Chung: Elementary Probability Theory with Stochastic Processes. Third edition.
Cox/Little/O'Shea: Ideals, Varieties, and Algorithms. Second edition.
Croom: Basic Concepts of Algebraic Topology.
Curtis: Linear Algebra: An Introductory Approach. Fourth edition.
Devlin: The Joy of Sets: Fundamentals of Contemporary Set Theory. Second edition.
Dixmier: General Topology.
Driver: Why Math?
Ebbinghaus/Flum/Thomas: Mathematical Logic. Second edition.
Edgar: Measure, Topology, and Fractal Geometry.
Elaydi: An Introduction to Difference Equations. Second edition.
Exner: An Accompaniment to Higher Mathematics.
Exner: Inside Calculus.
Fine/Rosenberger: The Fundamental Theory of Algebra.
Fischer: Intermediate Real Analysis.
Flanigan/Kazdan: Calculus Two: Linear and Nonlinear Functions. Second edition.
Fleming: Functions of Several Variables. Second edition.
Foulds: Combinatorial Optimization for Undergraduates.
Foulds: Optimization Techniques: An Introduction.
Franklin: Methods of Mathematical Economics.
Frazier: An Introduction to Wavelets Through Linear Algebra.
Gamelin: Complex Analysis.
Gordon: Discrete Probability.
Hairer/Wanner: Analysis by Its History.
Readings in Mathematics.
Halmos: Finite-Dimensional Vector Spaces. Second edition.
Halmos: Naive Set Theory.
Hämmerlin/Hoffmann: Numerical Mathematics.
Readings in Mathematics.
Harris/Hirst/Mossinghoff: Combinatorics and Graph Theory.

(continued after index)

Klaus Jänich

Vector Analysis

Translated by Leslie Kay

With 108 Illustrations

Springer

Klaus Jänich
NWF-I Mathematik
Universität Regensburg
D-93040 Regensburg
Germany

Leslie Kay (*Translator*)
Department of Mathematics
Virginia Tech
Blacksburg, VA 24061-0123
USA

Mathematics Subject Classification (2000): 57-01, 57Rxx

Library of Congress Cataloging-in-Publication Data
Jänich, Klaus.
[Vektoranalysis. English]
Vector analysis / Klaus Jänich.
p. cm. – (Undergraduate texts in mathematics)
Includes bibliographical references and index.
ISBN 978-1-4419-3144-3 ISBN 978-1-4757-3478-2 (eBook)
DOI 10.1007/978-1-4757-3478-2
1. Vector analysis. I. Title. II. Series.
QA433.J3613 2000
515′.63–dc21 99-16555

Printed on acid-free paper.

This book is a translation of the second German edition of *Vektoranalysis*, by Klaus Jänich, Springer-Verlag, Heidelberg, 1993.

Production managed by Jenny Wolkowicki; manufacturing supervised by Erica Bresler.
Typeset by Integre Technical Publishing Co., Inc., Albuquerque, NM.

9 8 7 6 5 4 3 2 1

ISBN 978-1-4419-3144-3 SPIN 10696691

Preface to the English Edition

Addressing the English-speaking readers of this book, I should state who I imagine those readers are. The preface to the first German edition was written for students in a different academic system, and the description I gave there doesn't apply directly. Should we, in this global age, have more compatibility in academic education? There is a debate going on now in Germany about whether we should introduce the bachelor's degree, or "Bakkalaureus" as some would call it, so that our system can be more easily compared with those abroad. Difficult questions! But it has been observed that whatever the academic system, students of the same *age* have about the same level of knowledge and sophistication. Therefore I can simply say that this is a book for twenty-year-old students.

This book is about manifolds, differential forms, the Cartan derivative, de Rham cohomology, and the general version of Stoke's theorem. This theory contains classical vector analysis, with its gradient, curl, and divergence operators and the integral theorems of Gauss and Stokes, as a special case. But since the student may not immediately recognize this fact, some care is given to the translation between these two mathematical languages.

Speaking of translation, I would like to acknowledge the excellent work of Leslie Kay in translating the German text into English. We have exchanged detailed e-mail messages throughout the translation process, discussing mathematics and subtleties of language. While I was using the opportunity of this English edition to eliminate all the typos and mistakes I knew of in the present German edition, Dr. Kay initiated many additional improvements. I wish to thank her for all the care she has devoted to the book.

Langquaid, Germany Klaus Jänich
October 2000

Preface to the First German Edition

An elegant author says in two lines what takes another a full page. But if a reader has to mull over those two lines for an hour, while he could have read and understood the page in five minutes, then—for this particular reader—it was probably not the right *kind* of elegance. It all depends on who the readers are.

Here I am writing for university students in their second year, who know nothing yet about manifolds and such things, but can feel quite satisfied if they have a good overall understanding of the differential and integral calculus of one and several variables. I ask other possible readers to be patient from time to time. Of course, I too would like to combine both kinds of elegance, but when that doesn't work I don't hesitate to throw line-saving elegance overboard and stick to time-saving elegance. At least that's my intention!

Introductory textbooks are usually meant "to be used in conjunction with lectures," but even this purpose is better served by a book that can be understood on its own. I have made an effort to organize the book so that you can work through it on a desert island, assuming you take your lecture notes from your first two semesters along and—in case those lectures didn't include topology—a few notes on basic topological concepts.

Since discussion partners are sometimes hard to find on desert islands, I have included *tests*, which I would like to comment on now. Some people disapprove of multiple-choice tests on principle because they think putting check marks in boxes is primitive and unworthy of a mathematician. It's hard to argue with that! Actually, some of my test questions are so utterly and obviously simple that they'll give you—a healthy little scare when you find you can't answer them after all. But many of them are hard, and resisting the specious arguments of the wrong answers takes some firmness. The tests should be taken seriously as a training partner for the reader who is alone with the book. By the way, there is at least one right answer in each set of three, but there may be several.

Now I won't describe the book any further—it's in front of you, after all—but will turn instead to the pleasant duty of looking back when the work is done and gratefully acknowledging the many kinds of help I received.

Martina Hertl turned the manuscript into TEX, and Michael Prechtel was always there with his advice and support as a TEX wizard. I received useful macros from Martin Lercher as well as from the publisher, and I was one of the first to use diagram.tex, developed by Bernhard Rauscher, for the diagrams. My students Robert Bieber, Margarita Kraus, Martin Lercher, and Robert Mandl expertly proofread the next to the last version of the book. I am very grateful for all their help.

Regensburg, Germany
June 1992

Klaus Jänich

Contents