

Introduction to the Theory of Nonlinear Optimization

Johannes Jahn

Introduction to the Theory of Nonlinear Optimization

Third Edition

With 31 Figures

 Springer

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Library of Congress Control Number: 2006938674

ISBN 978-3-540-49378-5 Springer Berlin Heidelberg New York
ISBN 978-3-540-61407-4 Second Edition Springer Berlin Heidelberg New York

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Production: LE-TeX Jelonek, Schmidt & Vöckler GbR, Leipzig
Cover-design: Erich Kirchner, Heidelberg

SPIN 11932048 42/3100YL - 5 4 3 2 1 0 Printed on acid-free paper

To Claudia and Martin

Preface

This book presents an application-oriented introduction to the theory of nonlinear optimization. It describes basic notions and conceptions of optimization in the setting of normed or even Banach spaces. Various theorems are applied to problems in related mathematical areas. For instance, the Euler-Lagrange equation in the calculus of variations, the generalized Kolmogorov condition and the alternation theorem in approximation theory as well as the Pontryagin maximum principle in optimal control theory are derived from general results of optimization.

Because of the introductory character of this text it is not intended to give a complete description of all approaches in optimization. For instance, investigations on conjugate duality, sensitivity, stability, recession cones and other concepts are not included in the book.

The bibliography gives a survey of books in the area of nonlinear optimization and related areas like approximation theory and optimal control theory. Important papers are cited as footnotes in the text.

This third edition is an enlarged and revised version containing an additional chapter on extended semidefinite optimization and an updated bibliography.

I am grateful to S. Geuß, S. Gmeiner, S. Keck, Prof. Dr. E.W. Sachs and H. Winkler for their support, and I am especially indebted to D.G. Cunningham, Dr. G. Eichfelder, Dr. F. Hettlich, Dr. J. Klose, Prof. Dr. E.W. Sachs, Dr. T. Staib and Dr. M. Stingl for fruitful discussions.

Erlangen, September 2006

Johannes Jahn

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