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Lectures in Abstract Algebra

I. Basic Concepts

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TO

MY WIFE

PREFACE

The present volume is the first of three that will be published under the general title *Lectures in Abstract Algebra.* These volumes are based on lectures which the author has given during the past ten years at the University of North Carolina, at The Johns Hopkins University, and at Yale University. The general plan of the work is as follows: The present first volume gives an introduction to abstract algebra and gives an account of most of the important algebraic concepts. In a treatment of this type it is impossible to give a comprehensive account of the topics which are introduced. Nevertheless we have tried to go beyond the foundations and elementary properties of the algebraic systems. This has necessitated a certain amount of selection and omission. We feel that even at the present stage a deeper understanding of a few topics is to be preferred to a superficial understanding of many.

The second and third volumes of this work will be more specialized in nature and will attempt to give comprehensive accounts of the topics which they treat. Volume II will bear the title *Linear Algebra* and will deal with the theory of vector spaces. Volume III, *The Theory of Fields and Galois Theory*, will be concerned with the algebraic structure of fields and with valuations of fields.

All three volumes have been planned as texts for courses. A great many exercises of varying degrees of difficulty have been included. Some of these perhaps rate stars, but we have felt that the disadvantages of the system of starring difficult exercises outweigh its advantages. A few sections have been starred (notation: *1) to indicate that these can be omitted without jeopardizing the understanding of subsequent material.

We are indebted to a great many friends for helpful criticisms and encouragement during the course of preparation of this volume. Professors A. H. Clifford, G. Hochschild and R. E. Johnson, Drs. D. T. Finkbeiner and W. H. Mills have read parts of the manuscript and given us useful suggestions for improving it. Drs. Finkbeiner and Mills have assisted with the proofreading. I take this opportunity to offer my sincere thanks to all of these men.

N. J.

New Haven, Conn.
January 22, 1951

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