# Finite-Dimensional Division Algebras over Fields

Nathan Jacobson

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Springer

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Library of Congress Cataloging-in-Publication Data

Jacobson, Nathan, 1910-1999
Finite-dimensional division algebras over fields / Nathan Jacobson.
p. cm.
Includes bibliographical references (p. 275-280).
ISBN 3-540-57029-2 (Berlin : hardcover : alk. paper)
1. Division algebras. 2. Fields (Algebra) I. Title.
QA247.45.J33 1996
512'.24--dc20 96-31625
CIP

Corrections of the 1st edition (1996) carried out on behalf of
N. Jacobson (deceased) by Prof. P.M. Cohn (UC London, UK)

ISBN 978-3-540-57029-5 e-ISBN 978-3-642-02429-0
DOI 10.1007/978-3-642-02429-0
Springer Heidelberg Dordrecht London New York

Mathematics Subject Classification (1991): 13-XX, 16-XX, 17-XX



*Cover design*: WMXDesign GmbH, Heidelberg

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

# PREFACE

These algebras determine, by the Wedderburn Theorem, the semi-simple finite dimensional algebras over a field. They lead to the definition of the Brauer group and to certain geometric objects, the Brauer-Severi varieties.

We shall be interested in these algebras which have an involution. Algebras with involution arose first in the study of the so-called "multiplication algebras of Riemann matrices". Albert undertook their study at the behest of Lefschetz. He solved the problem of determining these algebras. The problem has an algebraic part and an arithmetic part which can be solved only by determining the finite dimensional simple algebras over an algebraic number field. We are not going to consider the arithmetic part but will be interested only in the algebraic part. In Albert's classical book (1939), both parts are treated. A quick survey of our Table of Contents will indicate the scope of the present volume.

The largest part of our book is the fifth chapter which deals with involutorial simple algebras of finite dimension over a field.

Of particular interest are the Jordan algebras determined by these algebras with involution. Their structure is determined and two important concepts of these algebras with involution are the universal enveloping algebras and the reduced norm. Of great importance is the concept of isotopy. There are numerous applications of these concepts, some of which are quite old.

In preparing this volume we have been assisted by our friends, notably Jean-Pierre Tignol and John Faulkner. Also, I am greatly indebted to my secretary, Donna Belli, and to my wife, Florie. I wish to thank all of them for their help.

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