

Cambridge University Press

0521556554 - A First Course in the Numerical Analysis of Differential Equations

Arieh Iserles

Frontmatter

[More information](#)

---

---

# A First Course in the Numerical Analysis of Differential Equations

---

ARIEH ISERLES

*Department of Applied Mathematics and Theoretical Physics  
University of Cambridge*



**CAMBRIDGE**  
**UNIVERSITY PRESS**

Cambridge University Press

0521556554 - A First Course in the Numerical Analysis of Differential Equations

Arieh Iserles

Frontmatter

[More information](#)

CAMBRIDGE UNIVERSITY PRESS

Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press

The Edinburgh Building, Cambridge CB2 2RU, UK

Published in the United States of America by Cambridge University Press, New York

[www.cambridge.org](http://www.cambridge.org)

Information on this title: [www.cambridge.org/9780521553766](http://www.cambridge.org/9780521553766)

© Cambridge University Press 1996

This publication is in copyright. Subject to statutory exception  
and to the provisions of relevant collective licensing agreements,  
no reproduction of any part may take place without  
the written permission of Cambridge University Press.

First published 1996

Reprinted 1998, 2000, 2002, 2004(with corrections)

*A catalogue record for this publication is available from the British Library*

ISBN-13 978-0-521-55376-6 hardback

ISBN-10 0-521-55376-8 hardback

ISBN-13 978-0-521-55655-2 paperback

ISBN-10 0-521-55655-4 paperback

Transferred to digital printing 2006

---



---

## *Contents*

---

Preface	xi
Flowchart of contents	xvii
<b>I Ordinary differential equations</b>	<b>1</b>
<b>1 Euler's method and beyond</b>	<b>3</b>
1.1 Ordinary differential equations and the Lipschitz condition . . . . .	3
1.2 Euler's method . . . . .	4
1.3 The trapezoidal rule . . . . .	8
1.4 The theta method . . . . .	13
Comments and bibliography . . . . .	14
Exercises . . . . .	15
<b>2 Multistep methods</b>	<b>19</b>
2.1 The Adams method . . . . .	19
2.2 Order and convergence of multistep methods . . . . .	21
2.3 Backward differentiation formulae . . . . .	26
Comments and bibliography . . . . .	29
Exercises . . . . .	31
<b>3 Runge–Kutta methods</b>	<b>33</b>
3.1 Gaussian quadrature . . . . .	33
3.2 Explicit Runge–Kutta schemes . . . . .	37
3.3 Implicit Runge–Kutta schemes . . . . .	41
3.4 Collocation and IRK methods . . . . .	42
Comments and bibliography . . . . .	47
Exercises . . . . .	50
<b>4 Stiff equations</b>	<b>53</b>
4.1 What are stiff ODEs? . . . . .	53
4.2 The linear stability domain and A-stability . . . . .	56
4.3 A-stability of Runge–Kutta methods . . . . .	59
4.4 A-stability of multistep methods . . . . .	63
Comments and bibliography . . . . .	68
Exercises . . . . .	70

<b>5 Error control</b>	<b>73</b>
5.1 Numerical software <i>vs</i> numerical mathematics . . . . .	73
5.2 The Milne device . . . . .	75
5.3 Embedded Runge–Kutta methods . . . . .	81
Comments and bibliography . . . . .	86
Exercises . . . . .	89
<b>6 Nonlinear algebraic systems</b>	<b>91</b>
6.1 Functional iteration . . . . .	91
6.2 The Newton–Raphson algorithm and its modification . . . . .	95
6.3 Starting and stopping the iteration . . . . .	98
Comments and bibliography . . . . .	100
Exercises . . . . .	101
<b>II The Poisson equation</b>	<b>103</b>
<b>7 Finite difference schemes</b>	<b>105</b>
7.1 Finite differences . . . . .	105
7.2 The five-point formula for $\nabla^2 u = f$ . . . . .	112
7.3 Higher-order methods for $\nabla^2 u = f$ . . . . .	123
Comments and bibliography . . . . .	128
Exercises . . . . .	131
<b>8 The finite element method</b>	<b>135</b>
8.1 Two-point boundary value problems . . . . .	135
8.2 A synopsis of FEM theory . . . . .	147
8.3 The Poisson equation . . . . .	155
Comments and bibliography . . . . .	163
Exercises . . . . .	165
<b>9 Gaussian elimination for sparse linear equations</b>	<b>169</b>
9.1 Banded systems . . . . .	169
9.2 Graphs of matrices and perfect Cholesky factorization . . . . .	174
Comments and bibliography . . . . .	179
Exercises . . . . .	182
<b>10 Iterative methods for sparse linear equations</b>	<b>185</b>
10.1 Linear, one-step, stationary schemes . . . . .	185
10.2 Classical iterative methods . . . . .	193
10.3 Convergence of successive over-relaxation . . . . .	204
10.4 The Poisson equation . . . . .	214
Comments and bibliography . . . . .	219
Exercises . . . . .	224

<b>11 Multigrid techniques</b>	<b>227</b>
11.1 In lieu of a justification . . . . .	227
11.2 The basic multigrid technique . . . . .	234
11.3 The full multigrid technique . . . . .	238
11.4 Poisson by multigrid . . . . .	240
Comments and bibliography . . . . .	242
Exercises . . . . .	243
<b>12 Fast Poisson solvers</b>	<b>245</b>
12.1 TST matrices and the Hockney method . . . . .	245
12.2 The fast Fourier transform . . . . .	249
12.3 Fast Poisson solver in a disc . . . . .	256
Comments and bibliography . . . . .	262
Exercises . . . . .	264
<b>III Partial differential equations of evolution</b>	<b>267</b>
<b>13 The diffusion equation</b>	<b>269</b>
13.1 A simple numerical method . . . . .	269
13.2 Order, stability and convergence . . . . .	275
13.3 Numerical schemes for the diffusion equation . . . . .	282
13.4 Stability analysis I: Eigenvalue techniques . . . . .	287
13.5 Stability analysis II: Fourier techniques . . . . .	292
13.6 Splitting . . . . .	297
Comments and bibliography . . . . .	301
Exercises . . . . .	303
<b>14 Hyperbolic equations</b>	<b>307</b>
14.1 Why the advection equation? . . . . .	307
14.2 Finite differences for the advection equation . . . . .	314
14.3 The energy method . . . . .	325
14.4 The wave equation . . . . .	327
14.5 The Burgers equation . . . . .	333
Comments and bibliography . . . . .	338
Exercises . . . . .	342
<b>Appendix: Bluffer's guide to useful mathematics</b>	<b>347</b>
Bluffer's guide to useful mathematics	<b>347</b>
A.1 Linear algebra . . . . .	348
A.1.1 Vector spaces . . . . .	348
A.1.2 Matrices . . . . .	349
A.1.3 Inner products and norms . . . . .	352
A.1.4 Linear systems . . . . .	354
A.1.5 Eigenvalues and eigenvectors . . . . .	357
Bibliography . . . . .	359
A.2 Analysis . . . . .	359
A.2.1 Introduction to functional analysis . . . . .	359

Cambridge University Press

0521556554 - A First Course in the Numerical Analysis of Differential Equations

Arieh Iserles

Frontmatter

[More information](#)

x

*Contents*

A.2.2	Approximation theory . . . . .	362
A.2.3	Ordinary differential equations . . . . .	364
	Bibliography . . . . .	365
<b>Index</b>		<b>367</b>