

David Colton Rainer Kress

Inverse Acoustic and Electromagnetic Scattering Theory



Springer-Verlag Berlin Heidelberg GmbH

David Colton
Department of Mathematical Sciences
University of Delaware
Newark, DE 19716, USA

Rainer Kress
Institut für Numerische und Angewandte Mathematik
Universität Göttingen
Lotzestraße 16–18
W-3400 Göttingen, Fed. Rep. of Germany

Editors

F. John	J. E. Marsden	L. Sirovich
Courant Institute of Mathematical Sciences	Department of Mathematics	Division of Applied Mathematics
New York University	University of California	Brown University
New York, NY 10012	Berkeley, CA 94720	Providence, RI 02912
USA	USA	USA

Mathematics Subject Classification (1991):

35J05, 35P25, 35Q60, 35R25, 35R30, 45A05, 65M30, 65R20, 65R30, 76Q05, 78A45

With 6 Illustrations

ISBN 978-3-662-02837-7 ISBN 978-3-662-02835-3 (eBook)
DOI 10.1007/978-3-662-02835-3

Library of Congress Cataloging-in-Publication Data. Colton, David L. Inverse acoustic and electromagnetic scattering theory / David Colton, Rainer Kress. p. cm. – (Applied mathematical sciences; v. 93) Includes bibliographical references and index.

1. Sound-waves-Scattering. 2. Electromagnetic waves-Scattering. 3. Inverse scattering transform. I. Kress, Rainer, 1941-. II. Title. III. Series: Applied mathematical sciences Springer-Verlag Berlin Heidelberg GmbH; v. 93. QA1.A647 vol. 93 [QC243.3.S3] 510 s-dc20 [534'.2] 92-16437

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilm or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer-Verlag. Violations are liable for prosecution under the German Copyright Law.

© Springer-Verlag Berlin Heidelberg 1992

Originally published by Springer-Verlag Berlin Heidelberg New York in 1992.

Softcover reprint of the hardcover 1st edition 1992

The use of general descriptive names, registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

Typesetting: Camera ready by authors

41/3140-5 4 3 2 1 0 – Printed on acid-free paper

Table of Contents

1	Introduction	1
1.1	The Direct Scattering Problem	2
1.2	The Inverse Scattering Problem	7
2	The Helmholtz Equation	12
2.1	Acoustic Waves	12
2.2	Green's Theorem and Formula	15
2.3	Spherical Harmonics	20
2.4	Spherical Bessel Functions	27
2.5	The Far Field Mapping	31
3	Direct Acoustic Obstacle Scattering	37
3.1	Single- and Double-Layer Potentials	38
3.2	Scattering from a Sound-Soft Obstacle	45
3.3	The Reciprocity Relation	53
3.4	The Two-Dimensional Case	63
3.5	On the Numerical Solution in $\mathbb{R}^2$	66
3.6	On the Numerical Solution in $\mathbb{R}^3$	77
4	Ill-Posed Problems	84
4.1	The Concept of Ill-Posedness	85
4.2	Regularization Methods	86
4.3	Singular Value Decomposition	88
4.4	Tikhonov Regularization	96
4.5	Nonlinear Operators	101
5	Inverse Acoustic Obstacle Scattering	104
5.1	Uniqueness	105
5.2	Physical Optics Approximation	112
5.3	The Direct Approach to the Inverse Problem	113
5.4	Approximation of the Scattered Field	127
5.5	Superposition of the Incident Fields	137

X Table of Contents

6	The Maxwell Equations	146
6.1	Electromagnetic Waves	147
6.2	Green's Theorem and Formula	148
6.3	Vector Potentials	158
6.4	Scattering from a Perfect Conductor	165
6.5	Vector Wave Functions	170
6.6	The Reciprocity Relation	178
7	Inverse Electromagnetic Obstacle Scattering	188
7.1	Uniqueness	188
7.2	Continuous Dependence on the Boundary	192
7.3	Approximation of the Scattered Field	196
7.4	Superposition of the Incident Fields	199
8	Acoustic Waves in an Inhomogeneous Medium	204
8.1	Physical Background	205
8.2	The Lippmann-Schwinger Equation	207
8.3	The Unique Continuation Principle	211
8.4	Far Field Patterns	215
8.5	The Analytic Fredholm Theory	222
8.6	Transmission Eigenvalues	227
8.7	Numerical Methods	234
9	Electromagnetic Waves in an Inhomogeneous Medium	238
9.1	Physical Background	239
9.2	Existence and Uniqueness	240
9.3	Far Field Patterns	245
9.4	The Spherically Stratified Dielectric Medium	247
9.5	The Exterior Impedance Boundary Value Problem	252
10	The Inverse Medium Problem	260
10.1	The Inverse Medium Problem for Acoustic Waves	260
10.2	A Uniqueness Theorem	262
10.3	A Dual Space Method	266
10.4	A Modified Dual Space Method	270
10.5	The Inverse Medium Problem for Electromagnetic Waves	275
10.6	Numerical Examples	286
	References	293
	Index	303