

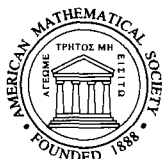
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**Real Analysis —
With an Introduction
to Wavelet Theory**

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